An Empirical Investigation of Long-Run Risks Models using Stock and Derivative Data*

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Abstract

This paper systematically calibrates a series of long-run risks (LRR) models to describe market aggregate behavior. We show that the fitted models can simultaneously explain many central features in asset pricing such as high equity premiums, long-range dependence in stock volatility, and high variance risk premiums – the difference between the option-implied variance and the expected realized variance of the underlying equity index. Our models improve on standard LRR models by adding two key features: jumps in the fundamental state variables and a two-factor volatility structure. Both features are crucial to successfully explain the key stylized facts of the U.S. stock and option markets. Moreover, the fundamental jump risk constitutes up to 45% of the equity premium, which reveals an important connection between equity excess returns and the variance risk premium. Finally, contributing to the growing literature of stock-return predictability by the variance risk premium, we find that the test results depend on the sample period, how the variance risk premium is measured and how the regression test is conducted. When the variance risk premium is measured based on the HAR-RV (Heterogenous Autoregressive Model of the Realized Volatility) method, the regression statistics are imputed from the univariate VAR analysis, and the time period is from 1990 to 2007, the LRR models can fit the data quite well.

1 Introduction

The Long Run Risks (LRR) model, originally proposed by Bansal and Yaron (2004)(BY), has attracted a lot of interest recently. One of the few leading candidate models to explain high and volatile equity risk premiums and low and smooth risk free rates, the LRR model features

*The views presented in this paper are those of the author's and are not necessarily shared by the Bank of Canada.

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a small but persistent component in consumption and dividend growth rates (the so-called long-run risk factor), a representative agent who has Epstein-Zin-Weil type recursive utility, and a time-varying (and persistent) stochastic volatility in consumption and dividend growth processes.

Recently, there have been a lot of studies based on LRR models attempting to explain the co-existence of large equity and variance risk premiums. Variance risk premium refers to the difference between the risk-neutral and physical expected variances of stock returns. The two premiums are usually studied separately. But a general-equilibrium model such as the LRR model offers a unified approach to understanding the relationship between them (Bollerslev et al. 2010,Drechsler et al. 2009).

The main purpose of this paper is to provide more evidence of the strengths and limitations of the LRR model, especially in the context of understanding the relationship between the equity and variance risk premiums. To do so, we systematically calibrate a series of LRR-based models against a broad set of asset classes, including equity indices, short-term interest rates and variance swap rates. One particular difference between our study and previous studies is that we consider it an important matter to calibrate the model so that it can describe the persistence of stock realized volatility in the short and long run.

The main findings of this paper can be summarized as follows. First, we find that after certain extensions to the original BY model, the generalized LRR models can qualitatively explain the high variance risk premium and the long-range dependence in option-implied volatility.[1] Features such as jumps and two-volatility factor structures are essential for the model to achieve a good fit. Additionally, we compare two channels that can both explain the existence of large variance risk premiums. One channel involves jumps in the long-run risk factor and the other involves jumps in the volatility of the long-run risk factor. We find that the former channel is more effective than the latter in generating large variance risk premiums. However, the former channel will induce a low persistence in realized volatility, while the latter channel can maintain the high persistence reasonably well. Such comparisons would be difficult to make in previous studies because the persistence of stock volatility is not considered in the calibration process.

Secondly, we find that jump risks in the long-run component explain a significant fraction of the unconditional equity risk premium. For models with jumps in the long-run risk factor, the fraction can be as large as 45%; for models with jumps in the volatility of the long-run risk factor, the fraction can still be as high as 20%. Our results suggest a close connection between the variance risk premium and the equity risk premium because the variance risk premiums are almost solely contributed by jumps. This conclusion is consistent with the non-parametric,

[1] The option-implied volatility is defined as the square-root of the expected risk neutral variance. In practice, it is measured as the level of the VIX index from the CBOE(Chicago Board of Exchange).

high-frequency-based study of Bollerslev et al.(2010).

Thirdly, the variance risk premium's predictability of stock excess returns is closely examined. First of all, we find that the empirical association between the variance risk premium and excess returns clearly depends on what sample period is used, how the variance risk premium is measured and what regression method is used. Additionally, we find that including the second persistent volatility factor generally increases the predictability of stock returns by the variance risk premium. Finally, we find that our best calibrated LRR model can match a selected predictability pattern reasonably well.

Finally, we need to point out that there are several discrepancies between model and data that we cannot explain yet. The most important difference is the predictability of consumption and dividend growth by price-dividend ratios. Empirical evidence for consumption and dividend growth predictability is scarce; however, the calibrated models suggest strong predictability for both consumption and dividend. Furthermore, there is also a conflict between model and data about the predictability of the volatilities of stock returns, consumption, and dividend. Even though the models imply strong predictability for all of them, the data do not. To be fair to the LRR model, we clearly set up a high standard for it, so we view all these drawbacks as inspirations for future research rather than critiques.

This paper is closely related to a growing literature that links the equity risk premium and the variance risk premium in a general equilibrium framework. Both Bollerslev et al. (2009) and Zhou (2010) explore the variance risk premium's predictability for short-run stock returns. These two studies propose a recursive utility representative investor framework where the variance-of-variance of consumption growth serves as the driving source of the variance risk premium. Alternatively, Benzoni et al. (2010) show that jumps in the long-run component of consumption growth can have a large impact on the short maturity options' implied volatility skew.[2] Similarly, Drechsler and Yaron (2009) show that jumps in the long-run component can explain the large and time-varying variance risk premium, which can predict stock returns in the short run.[3]

In addition to these models, jumps in consumption volatility have also been proposed as a major channel for generating the variance risk premium. Eraker (2008) proposes a reduced form model by directly imposing jumps on the volatility processes. Shaliastovich (2009) considers a more structural LRR model in which investors face uncertainty when estimating the long-run consumption growth. The volatility of the long-run component therefore can be interpreted as the confidence interval for investors. The model in Shaliastovich (2009) is also related to Drechsler (2008), who finds that the time-varying model uncertainty of investors can explain

[2] They further show that the updating of beliefs on the probability of jumps can explain the sharp steepening in the volatility skew following the 1987 stock market crash.

[3] Bollerslev et al. (2009) omit the LRR component but show that under the recursive utility framework, the variance-of-variance of consumption growth can explain the dynamic behavior of variance risk premium.

the variance risk premium.

Following these studies, we adopt a classic recursive utility formula and a reduced-form model of consumption volatility. In some models, the coefficient describing the long-run component volatility is different from the consumption volatility. The former factor can be loosely interpreted as the investor's subjective uncertainty of long-run consumption growth, the latter corresponds to the volatility of consumption growth in the short run. In other models, the long-run component and the total consumption share the same volatility factor, while that volatility factor mean-reverts to another stochastic volatility factor. This structure is proposed in Duffie et al. (2000).

Our calibration explores the conditional information from option markets in estimating the LRR model, similar strategies have been adopted in Shaliastovich (2009) and Eraker (2008). Shaliastovich (2008) introduces cross-sectional option prices to extract the latent state variable. Cross-sectional option data certainly contain rich information on the investors' attitudes towards market risk, yet individual option pricing, especially in the in-the-money(ITM) or deep out-of-the-money(OTM) ranges, can be subject to large biases due to possible model-pricing errors. A better alternative, which is adopted in our model, is to use variance swap rates that are constructed in a "model-free" way.

Our work is also inspired by several studies that emphasize the importance of including a second volatility factor in stock volatility (Chernov et al. 2003, Chacko et al. 2003, Engle et al. 1999). In the option literature, Bates (2000) and Duffie et al. (2000) suggest that multiple volatility factors can be important in explaining the skew and term structure of option-implied volatility. In the LRR literature, there have been a few studies that use a two-factor structure in their models, such as Drechsler and Yaron (2009), Shaliastovich (2009), Bansal and Shaliastovich (2009), Bollerslev et al. (2009), and Zhou and Zhu (2009). We adopt the two-factor structure mainly to enable the model to describe the long-memory feature in the model, while the previous studies fail to address this concern.

The main body of this chapter is organized as follows: Section 2 introduces the model setup, equilibrium solution, calibration strategy, and data; Section 3 reports the results; Section 4 concludes.

2 Model

2.1 Model Setup

Preferences

Following the standard LRR literature, we assume that investors have Epstein-Zin-Weil preferences. Under the continuous-time framework, the recursive utility function is

$$U(t) = \{(1 - e^{-(\ln\delta)dt})C_t^{1-\frac{1}{\psi}} + (e^{-(\ln\delta)dt})E_t(U(t+dt)^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\}^{\frac{1}{1-1/\psi}}, \tag{1}$$

where $\ln\delta$ is the compound discount rate, γ is the degree of risk aversion, and ψ represents the elasticity of intertemporal substitution (EIS).

For the conventional CRRA power utility function, the risk aversion coefficient γ is restricted to be the reciprocal of the EIS coefficient ψ. It implies that investors require no premium for facing uncertainties in consumption growth. When $\gamma < \frac{1}{\psi}$, investors prefer late resolution of uncertainty in consumption growth. When $\gamma > \frac{1}{\psi}$, which is the case for a typical LRR model setup, investors prefer early resolution of uncertainties in consumption growth. The level of ψ is also critical in deciding investors' behavior towards the consumption growth path. When $\psi > 1$, the LRR model implies that when the long-run consumption growth rate gets higher, the value of the consumption-claimed asset increases.

The continuous-time dynamics of the log of the Intertemporal Marginal Rate of Substitution(IMRS) m_t is close to its discrete-time analog, i.e.

$$dm_t = \theta\ln\delta dt - \frac{\theta}{\psi}dc_t - (1-\theta)dr_{c,t}, \tag{2}$$

where $\theta = (1-\gamma)/\,(1-1/\psi)$, dc_t is the growth rate of log consumption. $dr_{c,t}$ represents the log of the instantaneous return of an asset which is a claim on the consumption steam:

$$dr_{c,t} = \ln\frac{P_{t+dt}^{con} + C_{t+dt}}{P_t^{con}}, \tag{3}$$

where P_t^{con} can be interpreted as investors' aggregate wealth at time t.

Analogously, the log of the instantaneous return of an asset that is a claim on the dividend stream is defined as $dr_{d,t}$, where $dr_{d,t} = \ln\frac{P_{t+dt}+D_{t+dt}}{P_t}$represents the market return on dividend-claim assets.

Under the standard Campbell-Shiller log-linearization approximation, both $dr_{c,t}$ and $dr_{d,t}$ are expressed as linear functions of the log wealth-consumption ratio v_t^c and the log price-dividend ratio v_t^d, i.e.

$$dr_{c,t} = k_0 dt + k_1 dv_t^c - (1-k_1)v_t^c dt + d\log C_t \tag{4}$$

$$dr_{d,t} = k_{0d} dt + k_{1d} dv_t^d - (1-k_{1d})v_t^d dt + d\log D_t, \tag{5}$$

where $v_t^c = \log(P_t^{con}/C_t)$, and $v_t^d = \log(P_t/D_t)$. dv_t^c and dv_t^d represent the instantaneous changes of the two log ratios. k_1, k_0, k_{1d}, and k_{0d} are constants determined by the unconditional means of v_t^c and v_t^d. The detail of the derivations can be found in Eraker and Shaliastovich (2008) and Appendix A.

Economic Fundamentals

We follow the standard LRR literature to assume that investors make decsions under a continuous-time, real endowment economy where consumption and dividend are correlated but separate processes. The growth dynamics of $\log C_t$ and $\log D_t$ can be written as

$$d\log C_t = (\mu_C + x_t - \frac{1}{2}(\delta_{c1}V_t^f + (1-\delta_{c1})V_t^p)dt + \sqrt{\delta_{c1}V_t^f + (1-\delta_{c1})V_t^p}dW_{c,t} \tag{6}$$

$$\begin{aligned} d\log D_t &= (\mu_D + \phi_D x_t - \frac{1}{2}\varphi_d^2(\delta_{c1}V_t^f + (1-\delta_{c1})V_t^p)dt + \\ &\quad \varphi_d\sqrt{\delta_{c1}V_t^f + (1-\delta_{c1})V_t^p}(\rho_{dc}dW_{c,t} + \sqrt{1-\rho_{dc}^2}dW_{d,t}) \end{aligned} \tag{7}$$

where μ_C and μ_D are the unconditional growth rates of the log consumption and log dividend respectively; V_t^f and V_t^p are two volatility factors (either of which might not be both present in certain models); $\frac{1}{2}(\delta_{c1}V_t^f + (1-\delta_{c1})V_t^p)$ and $\frac{1}{2}\varphi_d^2(\delta_{c1}V_t^f + (1-\delta_{c1})V_t^p)$ are adjustment terms for Jensen's inequality; x_t represents the small, persistent component embedded in the expected consumption and dividend growth, we will call it the "long-run risk factor" in the following sections; ϕ_D characterizes the sensitivity of dividend growth on x_t; the scaling factor φ_d is used to capture the higher volatility of the dividend relative to consumption; $dW_{c,t}$ and $dW_{d,t}$ are independent Brownian motions; ρ_{dc} characterizes the correlation betwen the two Brownian motion parts in consumption and dividend.

The latent state variable dyanmics follow stochastic processes:

$$dx_t = -\kappa_x x_t dt + \varphi_e\sqrt{V_t^f}dW_{x,t} + \xi_x dN_x \tag{8}$$

$$\begin{aligned} dV_t^f &= [\kappa_v^f(a_{f1}V_t^p + (1-a_{f1})\bar{V}_t^f - V_t^f)]dt + \\ &\quad \sigma_w^f\sqrt{V_t^f}(\rho_{xf}dW_{xt} + \sqrt{1-\rho_{xf}^2}dW_{v,t}^f) + \xi_{Vf}dN_{Vf} - E(\xi_{Vf})E(dN_{Vf}) \end{aligned} \tag{9}$$

$$dV_t^p = \kappa_v^p(\bar{V}_p - V_t^p)dt + \sigma_w^p\sqrt{V_t^p}dW_{v,t}^p \tag{10}$$

where κ_x,κ_v^f, and κ_v^p are mean-reverting coefficients, ρ_{xf} is the diffusion correlation between x_t and V_t^f, φ_e, σ_w^f and σ_w^p are diffusion parameters for the state variables, $\xi_x dN_x$ and $\xi_{Vf} dN_{Vf}$ represent jump processes, $E(\xi_{Vf})E(dN_{Vf})$ is the compensation term. Finally, a_{f1} controls the long-run mean of V_t^f, which is V_t^p when a_{f1} is 1.

Following Drechsler and Yaron (2009), we assume that the intensities of jumps in the long-run component follow Poisson distributions with time-varying intensities i.e.

$$\begin{aligned} Prob(dN_x &= 1|I_t) = l_{XV} V_t^f dt \\ Prob(dN_{Vf} &= 1|I_t) = l_V V_t^f dt \end{aligned} \tag{11}$$

where l_{XV} and l_V are constants. This reflects the state-dependent feature of the jump process, when high-volatility state indicates a high probability of a sudden change in the expected long-run consumption growth rate. This specification is similar to Drechsler et al. (2009) and Eraker (2008).

We assume the jump size of x_t to be a left-skewed Gamma distribution:

$$\xi_x \sim -\Gamma(\gamma_x, \frac{\mu_x}{\gamma_x}) + \mu_x \tag{12}$$

which means that x_t has many small positive jumps and a few large negative jumps, and the mean jump size is zero.

We also assume the jump size of the short-run volatility factor V_t^f to be right skewed Gamma distribution

$$\xi_V \sim \Gamma(\gamma_V, \frac{\mu_V}{\gamma_V}). \tag{13}$$

In both distributions, the γ_x and γ_V are shape parameters and are set to the standard value of 1 and μ_x and μ_V are scale parameters to characterize the magnitude of the jumps.

We investigate several different specifications of the LRR model with each set of $(\delta_{c1}, a_{f1}, l_{XV}, l_V)$ indicating one particular type of the model. When $a_{f1} = 1$, and jumps are excluded, the model becomes quite similar to that in Bansal and Yaron (2004). Since this model features stochastic volatility in both consumption and dividend processes and has only one volatility factor, we label it as the SV1F model. When we assume that the volatility factor V_t^f of the expected long-run growth rate x_t is mean-reverting to another time-varying volatility factor V_t^p, which also controls the diffusion term in consumption and dividend growth processes, the model now has two stochastic volatility factors and no jump, and we label it as the SV2F model.

Furthermore, similar to Benzoni et al. (2010) and Drechsler et al. (2008), we consider an extended version of the SV1F model with jumps in x_t, if the volatility factor of x_t is mean-reverting to a constant level, the model is labeld as the SVJ1F_X model. On the other hand, if the volatility factor of x_t is mean-reverting to another time-varying volatilitly factor, then the model is labeled as the SVJ1F_X_SM model. The definitions of the SVJ1F_V model and the SVJ1F_V_SM model are similar to the SVJ1F_X and SVJ1F_X_SM model respectively, except that there are jumps in the volatility factor of the long-run risk factor.

The most generalized model assumes that diffusion terms of consumption and dividend are controlled by the long-run volatility factor V_t^p, the diffusion term of the long-run component x_t is controlled by the short-run volatility factor V_t^f, and V_t^f mean-reverts to V_t^p. There are jumps for both the long-run risk factor x_t and the short-run volatility factor V_t^f. This model is labeled as the SVJ2F model.

All these specifications can be expressed in an affine form that was discussed by Eraker et al.(2008), where the economic fundamental variables $(\log C_t, \log D_t)$ and state variables (x_t, V_t^f, V_t^p) can be expressed as a vector Y_t

$$dY_t = (M + K'Y_t)dt + \Sigma(Y_t)dW_t + \xi_t \cdot dN_t, \tag{14}$$

where $M \in \mathbb{R}^{5\times1}$, $K \in \mathbb{R}^{5\times5}$, $\Sigma(Y_t) \times \Sigma(Y_t)' = h + \sum_i H_i Y_t$, and $(h, H) \in \mathbb{R}^{5\times5} \times \mathbb{R}^{5\times5\times5}$. At the same time, the jump intensity coefficient describing the Poisson distribution of dN_t can be written as $l(Y_t) = l_0 + l_1 Y_t$, where $(l_0, l_1) \in R^5 \times R^{5\times5}$. In our specification, we set l_0 all to be zero. As shown in Appendix A, such an affine representation greatly simplifies the equilibrium solution expression.

Under the risk-neutral measure, the dynamics of the state variables can be expressed as

$$dY_t^Q = (M^Q + K^Q Y_t)dt + \Sigma(Y_t)dW_t^Q + \xi_t^Q \cdot dN_t^Q. \tag{15}$$

The details of the transformation between the P and Q measures are shown in Appendix A.

Solving the Model

The model solution is based on solving the Euler equation

$$E_t[\exp(m_t + r_{i,t})] = 1,\ i \in \{c, d\} \tag{16}$$

where $r_{c,t}$ and $r_{d,t}$ are the instantaneous returns of assets with claims on consumption stream and dividend stream respectively from time t to $t + dt$.

As shown in Appendix A, we follow the standard guess-and-verify procedure to solve the Euler equation. Both the log wealth-consumption ratio v_t^c and the log price-dividend ratio v_t^d are affine functions of stationary state variables x_t, V_t^f, V_t^p. For $Y_t = [\log C_t, x_t, V_t^f, V_t^p, \log D_t]'$, the equilibrium log wealth-consumption and price-dividend ratios can therefore be written as:

$$\begin{aligned} v_t^c &= A + B'Y_t \\ v_t^d &= A_d + B_d'Y_t, \end{aligned} \tag{17}$$

where A, B, A_d, B_d are functions of k_1, k_0, k_{1d}, and k_{0d}. A and A_d are scalars and B and B' are vectors.[4] The affine structure of the solution process is valid under the Campbell-Shiller log-linearization approximation. As shown by Bansal et al. (2007b), as long as the EIS is not significantly higher than 2 (which is satisfiled in our study), this log-linear approximation yields a result quite close to the more accurate numerical solution to the Euler equation.

The real risk-free rate can also be expressed as an affine function of the state variables

$$r_t = \Phi_0 + \Phi_1' Y_t, \tag{18}$$

where Φ_0 is a scalar and Φ_1 is an $n \times 1$ vector. The calculation of Φ_0 and Φ_1 follows the procedure in Eraker (2008). Details are in Appendix A.

The instantaneous expected equity risk premium can be written using the formula suggested in Drechsler et al. (2009) as an affine function of the two volatility factors V_t^f and V_t^p:

$$\begin{aligned} & E_t(r_{m,t+dt} - r_{f,t}) + 0.5Var_t(r_{m,t+dt}) \\ = \; & -Cov_t(m_{t+dt} - E_t(m_{t+dt}), r_{m,t+dt} - E_t(r_{m,t+dt})) \end{aligned} \tag{19}$$

where both $m_{t+dt} - E_t(m_{t+dt})$ and $r_{m,t+dt} - E_t(r_{m,t+dt})$ are linear functions of continuous shocks $dW_{c,t}$,dW_{xt},dW_{vf}, dW_{vp}, and jump shocks $\xi_x dN_x - E(\xi_x dN_x)$ and $\xi_{vf} dN_{vf} - E(\xi_{vf} dN_{vf})$,i.e.

$$\begin{aligned} m_{t+dt} - E_t(m_{t+dt}) &= m_1 dW_{c,t} + m_2 dW_{xt} + \\ & \quad m_3 dW_{vf} + m_4 dW_{vp} + \\ & \quad m_5[\xi_x dN_x - E(\xi_x dN_x)] \\ & \quad + m_6[\xi_{vf} dN_{vf} - E(\xi_{vf} dN_{vf})] \end{aligned} \tag{20}$$

and

$$\begin{aligned} r_{m,t+dt} - E_t(r_{m,t+dt}) &= r_1 dW_{c,t} + r_2 dW_{xt} + \\ & \quad r_3 dW_{vf} + r_4 dW_{vp} + \\ & \quad r_5[\xi_x dN_x - E(\xi_x dN_x)] \\ & \quad + r_6[\xi_{vf} dN_{vf} - E(\xi_{vf} dN_{vf})] \end{aligned} \tag{21}$$

[4] Since k_1, k_0, k_{1d},k_{0d} are determined by unconditional means of the log wealth-consumption ratio v_t^c and log price-dividend ratio v_t^d, Equation (16) needs to be iteratively run until a consistent solution is reached.

where m_i and r_i are constants.

The instantaneous return can therefore be calculated,

$$E_t(r_{m,t+dt} - r_{f,t}) + 0.5Var_t(r_{m,t+dt}) = \sum_{i=1}^{4} m_i r_i + m_5 r_5 Var_t(\xi_x dN_x) \quad (22)$$
$$+m_6 r_6 Var_t(\xi_{vf} dN_{vf}) \quad (23)$$

where each term in the right side represents the risk premium demanded for each individual risk source.

As suggested in Duffie et al. (2000) and Eraker (2008), we can take advantage of the fact that log stock prices are affine functions of the state variables so the moment generating functions for both P and Q measures can be expressed in a semi-closed form:

$$\begin{aligned} \psi^i(u, Y_t, 0, T) &= E_0^i \exp(u \ln S_T) \quad (24) \\ &= e^{\alpha_i(u,T)+\beta_i(u,T)Y_t},\ i \in \{P, Q\}, \end{aligned}$$

where $\alpha_i(u,t)$ and $\beta_i(u,t)$ satisfies

$$\frac{\partial \beta_i}{\partial t} = K^{i'}\beta + \frac{1}{2}\beta' H \beta + l_1^{i'}(\varrho^i(\beta) - 1) \quad (25)$$

and

$$\frac{\partial \alpha_i}{\partial t} = M^{i'}\beta + \frac{1}{2}\beta' h \beta + l_0^{i'}(\varrho^i(\beta) - 1), \quad (26)$$

with the initial conditions of $\alpha_i(u,0) = 0$ and $\beta_i(u,0) = u$.

The one-month ahead conditional variances in risk-neutral and physical measures can therefore be written as

$$\begin{aligned} Var_t^i[\ln R_{t,t+1}^i] &= \frac{\partial^2 \ln \psi^i(u, Y_t, t, t+1)}{\partial u^2}|_{u=0} \\ &= \alpha_i''(0,1) + \beta_i''(0,1)Y_t^i, i \in P, Q. \quad (27) \end{aligned}$$

The expectation of stock return quadratic variations under both physical and risk-neutral measures can be approximately calculated as the integrated conditional variance, i.e.

$$\begin{aligned} E^i[QV_{t,t+1}^i] &\simeq E^i[\sum_{n=1}^{M} Var_{t+\frac{n}{M}}^i[\ln R_{t+\frac{n}{M},t+\frac{n+1}{M}}]] \quad (28) \\ &\simeq Var_t^i[\ln R_{t,t+1}], i \in P, Q. \quad (29) \end{aligned}$$

The risk-neutral expected variance corresponds to the short-term variance swap rate, which can be represented by the square of the VIX index provided by CBOE(Chicago Board Options

Exchange). The physical expected variance can be estimated based on the high-frequency S&P 500 future data. The advantage of using future instead of cash index data is that the former avoids the stale price issue at high frequencies. Hence the variance risk premium (VRP) is defined as the difference between the expected variance under the two measures; i.e.,

$$VRP = E^Q[QV_{t,t+1}^Q] - E^P[QV_{t,t+1}^P] \tag{30}$$

It is apparent that under the two-factor model, the variance risk premium is an affine function of the short-run and long-run volatility factors V_t^f and V_t^p, while under the one-factor model, it is an affine function of the single volatility factor V_t.

2.2 Data

The data used in this paper are constructed based on various sources. The annual consumption data are obtained from BEA (The Bureau of Economic Analysis) and defined as the sum of per capita services and non-durable goods. The dividend, stock price, and short-term interest rates are from CRSP(The Center of Research in Security Prices) and COMPUSTAT. The cash dividend is calculated based on the difference in CRSP value-weighted market returns including and excluding dividends. The total dividend is constructed by adding stock repurchases to the cash dividend series. The method to estimate stock repurchases is the same as that used in Boudoukh et al. (2007).[5] The dividend is then adjusted to real terms based on Consumption Price Index (CPI). The high-frequency realized variance data are based on S&P 500 future obtained from CME (Chicago Mercantile Exchange). The VIX data is obtained from CBOE(Chicago Board of Exchange). The CPI(Consumption Price Index) data are obtained from BEA. Part of the expected inflation is constructured based on SPF(The Survey of Professional Forecasters).

[insert Table 3 about here]

[insert Figure 1 about here]

The upper panel in Table 3 reports the annual consumption and dividend growth from 1951 to 2010. The squared brackets record the 90% bootstrap confidence intervals. For the purpose of comparison, we also include the cash dividend data. As can be seen, aggregate consumption follows a smooth process with an average annual growth rate of approximately 2%. The standard deviation is about 1%. The consumption process also displays a slightly negative skewness and positive excess kurtosis and a moderate first autocorrelation of 0.38. The two dividend measures have significantly different characteristics. The average sample

[5] Since stock repurchasing data are only available after 1971, we do not adjust dividends for the period before 1971. As shown in Boudoukh et al. (2007), before 1971, payouts to investors were mainly in the form of cash dividends.

growth rate of the repurchase-adjusted dividend is 2.6%, much higher than the growth rate of 1% of the cash dividend. This reflects the well-known trend that firms are increasingly using stock repurchases as substitutes for cash dividends for tax and other reasons in the last twenty years. Moreover, the repurchase-adjusted dividend is much more volatile, leptokurtic, and persistent than the cash dividend.

Asset pricing data (except high-frequency and VIX) are in monthly frequency from January 1951 to July 2010.[6] The nominal risk-free rate is the yields on the three-month T-Bill, which are provided by the Fama Risk-free Rate Data Set in CRSP. The price-dividend ratios are constructed based on the log of the ratio of the end-of-month S&P 500 index to the trailing 12 month dividends. The inclusion of stock repurchases significantly decreases the growth of price-dividend ratios in recent years. As seen in Figure 1, the log price-dividend ratio with repurchase adjustment displays a stable movement from 1951 to 2010. On the other hand, the log price-dividend ratio without adjustment displays a clear upward trend after 1990 that may be unstationary. The monthly excess returns are measured by substracting the nominal risk free rate from the returns of S&P 500 index plus the cash dividend yield.

[insert Figure 2 about here]

All the nominal quantities except for the risk-free rates are adjusted to real terms based on the *ex post* monthly inflation rate provided by the CRSP. The adjustment of nominal risk-free rates is mostly based on SPF(The Survey of Professional Forecasters) because of the significant measurement noise in the *ex post* inflation shock. For the 1951-1967 part of the sample, we use the smoothed trend of the realized inflation as the approximation of forecasted inflation; for the 1968-2010 part of the sample, we use the forecasted GDP inflation index as the approximation of the expected inflation rate.[7] The survey based on the GDP deflator provides a longer sample(since 1968) than what is available for the CPI inflation (available since 1981). For most of the postwar period, the GDP deflator tracks the consumer price index (CPI) quite closely, as can be seen in Figure 2.[8]

Before 1990, the monthly realized variance is calculated by summing the daily squared returns over a month. After 1990, the realized variance is calculated based on the 5-min intraday trades of S&P 500 futures. The realized variance is first construted over a trading day

[6] We started the sample period in 1951 because there was a large degree of uncertainty in the economy during the Depression and World War II. Monthly inflation was also extremely volatile during the period from 1946 to 1950, making real risk-free rate estimation quite difficult.

[7] The SPF asks professional forecasters to submit their forecasts at the beginning of the second month of each quarter. The result is published in the middle of the same month. The content of the survey is clearly affected by the numbers published by the BEA report at the end of the first month of each quarter. To simplify, we assume that the forecasts for the current quarter are mostly made at the end of the last quarter.

[8] In an unpublished study, we also compare the surveyed inflation rate of the GDP price index and the surveyed CPI price index from 1981 to 2010. The difference in mean is only 0.24% annually, and the standard deviation, skewness, and kurtosis between these two measures are close as well.

and then aggregated over a month. We choose using S&P 500 futures as it has fewer problems with stale prices than high-frequency S&P 500 index.

The middle panel of Table 3 reports the summary statistics for asset prices. As can be seen, the risk-free rate is quite low and smooth, the log price-dividend ratio is quite stable, the equity premium is as high as 6.70% annually from 1951 to 2010, and the realized variance of the equity return is much higher than that of the consumption, dividend and the risk free interest rate.

The bottom panel of Table 3 reports the summary statistics for the monthly realized variance, VIX^2, and the associated variance risk premiums from 1990 to 2010. The calculation of variance risk premiums involves measuring the expected physical realized variance of stock return. To do so, the HAR-RV approach (Corsi (2009), Andersen et al. (2007)) is adopted. This approach involves conducting an in-sample regression of $\log RV_t$ on the realized variance of past day, week, month, quarter, and half-year, in a monthly unit, expressed as

$$\begin{aligned}\log RV_{t,t+22} &= \beta_0 + \beta_1 \log RV_{t-1,t} + \beta_2 \log RV_{t-5,t} \\ &\quad + \beta_3 \log RV_{t-22,t} + \beta_4 \log RV_{t-132,t} + \epsilon_t,\end{aligned} \tag{31}$$

and then construcing the expected realized variance based on the regression. Studies in Corsi (2009) and Andersen et al. (2007) suggest that the simple HAR-RV approach is at least on par with a much more sophisticated volatility forecasting model, and sometimes even performs better.

The summary statistics for the variance risk premiums suggests that they are on average positive, with high volatility and significantly large right tails. This is not surprising given the reported huge VIX spikes during times of financial crisis. Another salient feature is that both the physical volatility and option-implied volatility display a non-trivial correlation even at lags as long as 12 months. This is consistent with the "long-memory" characteristics of volatility that have been confirmed by more rigorous statistical tests.

2.3 Calibration Strategy

To calibrate the model parameters, we choose a systematic strategy based on the Simulated Methods of Moments (SMM) (Duffie et al.1993 , Gourieroux et al.1996).[9] Here is the procedure for carrying out the SMM calibration.

Step 1: Collecting the target moments of the data. We split the moments into two parts, the first part are moments that are based on the data from 1951 to 2010, while the second part are moments based on the volatility data from 1990 to 2010. Specially, we write the vector of moments $\tilde{y}(t)$ as

$$\tilde{y}(t) = [\tilde{y}_1(t), \tilde{y}_2(t)]$$

[9] Part of my code is adapted from Fackler and Tastan (2008).

where

$$\tilde{y}_1(t) = \begin{pmatrix} R_{ft}, (p-d)_t, (\sqrt{RV})_t \\ [R_{ft} - E(R_{ft})]^2, [(\sqrt{RV})_t - E(\sqrt{RV})]^2 \\ [(p-d)_t - E(p-d)]^2 \\ [R_{ft} - E(R_{ft})][R_{f,t-1} - E(R_{ft})] \end{pmatrix}$$

and

$$\tilde{y}_2(t) = \begin{pmatrix} VIX_t, [VIX_t - E(VIX)]^2), \\ [VIX_t - E(VIX)][VIX_{t-1} - E(VIX)] \\ [VIX_t - E(VIX)][VIX_{t-6} - E(VIX)], \\ [VIX_t - E(VIX)][\frac{1}{3}\sum_{n=11}^{13} VIX_{t-n} - E(VIX)] \end{pmatrix},$$

the first set of moments $\tilde{y}_1(t)$ includes the first and second moments of the risk-free rate, the log price-dividend ratio, and the realized volatility together with the first autocovariance moment of the risk-free rate, while the second set of $\tilde{y}_2(t)$ includes the moments based on the VIX data including the lag-1, lag-6 and lag-12 autocovariances (the lag-12 autocovariances are estimated by averaging lag-11, lag-12, and lag-13 autocovariances).

To overcome the issue of the shorter periods of option data compared to other asset market data, a procedure from Singleton (2006) is adopted to construct an overidentification vector of $M_{OID}(m, \tilde{y}_t)$ which includes data from both periods. The estimated moments minimize the objective function

$$\tilde{m}(\tilde{y}_t^{data}) = \arg \min_{m=[m_1, m_2]} M_{OID}(m, \tilde{y}_t^{data})' W_T M_{OID}(m, \tilde{y}_t^{data}),$$

where $M_{OID}(m, \tilde{y}^{data}) = [\frac{1}{T_1}\sum_1^{T_1} \tilde{y}_1^{data}(t) - m_1, \frac{1}{T_2}\sum_{T_1+1}^{T_1+T_2} \tilde{y}_1^{data}(t) - m_1, \frac{1}{T_2}\sum_1^{T_2} \tilde{y}_2^{data}(t) - m_2]$ is the overidentification vector. The estimation of W_T follows a standard two-stage process. Table 4 reports the estimated moments based on our sample. The biggest change is that after adjustment, the average realized volatility and VIX are markably lower than the simple sample means. For example, the sample average of the realized volatility reported in Table 3 is $\sqrt{19.78\% \times 12} = 15.41\%$ and the sample average of the VIX is 21.84%. After the adjustment, the sample average of the realized volatility and the VIX are 13.86% and 18.11%, respectively. This is due to the fact that the volatility prior to 1990 is lower than that in the post-1990 period.

[insert Table 4 about here]

Step 2: For a given parameter set θ, the state variable dynamics are discretized and simulated and the corresponding asset prices such as the short rates and the price dividend ratios are calculated based on the equilibrium solution at each period. Each simulation contains a sample size of 10,000 months. In the simulation, the same proportion of the VIX data is ignored the same inference procedure is conducted as in the real data.[10]

Step 3: The overidentification vector $M_{OID}(\tilde{m}(\tilde{y}_t, \hat{y}_{simu}(\theta))$ is constructed and the optimal calibration θ^0 is the solution to minimize the criteria function

$$M_{OID}(\tilde{m}(\tilde{y}_t), \hat{y}_{simu}(\theta))' \Omega_T M_{OID}(\tilde{m}(\tilde{y}_t), \hat{y}_{simu}(\theta)).$$

Here the optimal weighting matrix of Ω_T is estimated using a Newey-West estimator and a Bartlett weighting scheme with a lag length of 10.

Step 4: To compare the quality of fit across different models, we calculate the overidentification J-statistics and adjust them based on the fact that the moments are obtained by simulation instead of analytial solution.The typical adjustment based on the ratio of the simulated data length (10,000 months) to the realized data length (about 720 months) is applied. Since the simulation length is more than 10 times that of the actual data length, the adjustment is fairly small.

3 Results

3.1 Calibrated Models

For each calibrated model, we split the parameters into two groups. The parameters in the first group are preset at values consistent with most of the previous LRR studies. These parameters include the subjective discount factor (δ), the unconditional consumption growth rate(μ_C), the unconditional dividend growth rate(μ_D), the loading factor of dividend growth on the long-run consumption growth(ϕ_d), the correlation coefficient between consumption and dividend shocks(ρ_{dc}), the correlation coefficient between the shocks of the long-run risk factor and its volatility (ρ_{xf}), the jump intensity parameters (l_{XV} and l_V), and the jump distribution parameters (γ_x and γ_V).

All the other parameters are chosen as the solution which optimizes the fit between the model and the selected moments of the sample data. Among the second group of parameters, the loading factor of the dividend growth on the volatility factor φ_d is chosen among discrete values of 5, 6.5, 8, 9.5, and 11. For each candidate value, we optimize the objective function and we choose a φ_d that enables the model to achieve the best fit.

[10] We use the Euler scheme to discretize the continuous time dynamics. The time interval is at a frequency of 1/20 of a month, roughly corresponding to a daily frequency. The simulated variables are then aggregated at a monthly level.To ensure that the discretizaton error is small, we compare the simulation result of 1/20 of a month and that of 1/40 of a month to find that results are quite similar. We also compare the simulated option price based on discretization of 1/40 of a month with one based on a semi-closed form, continuous time calculation, and the two prices are also quite close, again confirming the adequacy of my discretization choice. During the simulation, we start with the initial state variables at their unconditional means and then discard the first 1 million steps. This is a practical way of randomizing the initial condition so that the simulated process will be stationary.

[insert Table 5 about here]

Table 5 reports our calibration results for several specifications of the long-run risk models. Seven different models are considered here. The first two are the SV1F and SV2F models, introduced in the previous section; these two models are diffusion models with one or two volatility factors. The next two models are the SVJ1F_X and SVJ1F_V models. They are similar to the classic SV1F model except that we assume that there are jumps in either the long-run risk factor (SVJ1F_X) or the volatility of the long-run risk factor (SVJ1F_V). The next two models, the SVJ1F_X_SM and SVJ1F_V_SM models are extensions of the SVJ1F_X and SVJ1F_V models respectively. These two models assume that the long-run mean of the volatility factor V_f is not constant but follows a time-varying stochastic process (SM stands for "stochastic mean"). The last model is the most generalized model, in which we assume that there are two volatility factors V_t^f and V_t^p. The former controls the volatility of the long-run risk factor and the latter controls the volatility of consumption and dividend. At the same time, V_t^p is the long term mean of V_t^f. Not surprisingly, one would expect V_t^f to have a much higher mean-reverting speed than V_t^p.

The first panel in Table 5 reports the preset parameters for all models. Most of the parameters are the same across different models. One exception is the unconditional dividend growth rate μ_d, which is set at 0.002 in the SV1F and SV2F models and set at 0.0028 in the other models. However, μ_d is mainly used to fit the unconditional growth rates of dividends, and its impact on other moments is relatively small. Another parameter is the jump intensity l_V for the volatility factor. In the SVJ2F model, we set l_V at 4000, while in the SVJ1F_V and SVJ1F_V_SM models we set it at 500. As we will discuss later, exactly identifying the jump intensity is extremely hard given the data, the impact of different jump intensities can be offset by different jump size distributions. However, our experience with jump-in-volatility models is that a lower jump intensity is able to accommodate a larger jump size in volatility, which is necessary for jump-in-volatility models to fit data moments well.

The second panel in Table 5 presents optimized parameters for the model to achieve the best sample fit. Overall, the SV1F and SV2F models fit the data poorly with large J-statistics and the SVJ2F model achieves the best in-sample fit. Including jump process is crucial for the model to fit the moments and including the second volatility factor also matters. Given that everything else is equal, the fit for jump-in-volatility models is slightly worse than that of jump-in-long-run-risk models.

The table also suggests that the loading parameter φ_d, the risk aversion parameter γ, the EIS parameter ψ, the persistence of the long-run risk factor κ_x and the diffusion coefficients φ_e are more or less similar across different models. They are also similar to the values provided in previous studies. This similarity mostly comes from the requirement for the model to fit the first and second moments of risk free rates, price-dividend ratios, and stock volatility. The mean-reverting parameter κ_v^f of the volatility factor V_t^f varies across different models. But they are typically larger than the previously reported mean-reverting coefficient. This is because the goal of our model is to fit the persistence of expected variance at the monthly level instead of the annual level. The mean-reverting parameter κ_v^p for the volatility factor V_t^p, is as small as 0.01, corresponding to a half decaying period of up to 70 months. Similarly persistent volatility has also been found in Egloff et al. (2010) whose estimation is based on the term-structure of variance swap rates. When V_t^f and V_t^p are both present in the model, the diffusion parameter V_t^f is generally a magnitude larger than the diffusion parameter of V_t^p. Given the fact that both factors have similar long-term mean (in two-factor models, they are the same), it suggests that the volatility of V_t^f is much higher than that of V_t^p. However, when the model includs large jumps in volatility, the volatility of V_t^f is much smaller than other models. For example, in the SVJ1F_V_SM model, σ_w^f is 3.26×10^{-4}, quite similar to σ_w^p which is 2.73×10^{-4}.

[insert Table 6 about here]

Table 6 and Table 7 report moment matches between the calibrated models and the data. The model implied moments are medians of the 1000 simulations with each simulation spanning a period of 60 years. Long-term simulation, not reported here, suggests that these medians are also jointly close to the moments of a long-period simulation moments. The numbers in brackets correspond to the 5% and 95% quantiles of the finite sample simulation.

Table 6 reports the moment match of the SV1F and SV2F models. These two models achieve decent fits to the first and second moments of interest rates, price-dividend ratios, and historical stock volatility. But the model-implied risk-neutral volatility(VIX) is significantly lower than the sample mean. As a consequence, these two models both generate variance risk premiums that are one magnitude smaller than the sample. In addition, the autocorrelation of the expected risk neutral volatility (VIX) in the SV1F model decays quickly as lag increases while the SV2F model demonstrates a long range dependence in expected risk neutral volatility which is more consistent with the data. This is presumably why the SV2F model achieves a better overall fit to the data than the SV1F model.

Table 7 reports the moment match for the models including jumps. The same as Table 6, the value reported is the median of 1000 simulations of 60 years (the VIX only has 20 years of observations) and the brackets contain five and ninety-five percent quantiles.

Similar to the SV1F and SV2F models, all the jump models display remarkable in-sample fit for the first and second moments of risk free rates, price-dividend ratios, and historical volatility. When jump are included, all the models display significant variance risk premiums. Nevertheless, models with jumps in long-run risk factors generally generate higher variance risk premiums than models with jumps in volatility. Furthermore, all models generate smaller variance risk premiums than the sample data. Aside from possible model misspecification, it is partially due to that fact that our models are calibrated to fit the estimated unconditional moments based on long and short periods of data jointly rather than based on the short-period of data over the last 20 years.

[insert Table 7 about here]

As for the long-range dependence in risk-neutral volatilities, we find that only the SVJ1F_V_SM model generates satisfying long-range dependence. This seems to be surprising initially, as long-time simulation suggests that the SVJ2F, SVJ1F_V_SM and SVJ1F_X_SM models display autocorrelation close to 0.30 for the lag of 12 months, and the SVJ1F_X and SVJ1F_V models display negligible autocorrelation at such a long lag. Hence, it is likely that the statistics in the finite sample seem to deviate from that trend. In fact, in the SVJ1F_V model the 12-month autocorrelation in the VIX is as high as 0.27, even higher than 6-month autocorrelation, which is clearly counterfactual. The difference between the finite sample statistics and population statistics indicates that our simple estimation of the long lag autocorrlation in the VIX is subject to robustness issues. Although sample autocorrelation estimation may not be robust against the finite sample effect, the long memory feature of stock volatility has been demonstrated by more robust statistical models and against long period data. So any models that are short of long-memory characteristics are subject to model misspecification to some degree. Establishing a more robust sample statistics describing the feature of long-memory is desirable.

Another important moment match, though not considered when calibrating the model, is the persistence of the realized volatility. The cost of assuming too large a jump in the long-run risk factor is that the model does not display the GARCH effect. This is because when there is a jump in the long-run risk factor (including occasionally a large negative jump), the sudden price movement is not accompanied by a change in volatility factor. Consequently, the volatility of the future price movement is not affected by the jump in the long-run risk factor at all. For example, in the SVJ2F model, the model-implied autocorrelation for the realized volatility is 0.36, much smaller than the sample correlation of 0.63. On the other hand, the problem seems to be much smaller for the SVJ1F_X_SM and SVJ1F_V_SM models. This is partly due to the fact that these two models generate much smaller variance risk premiums than the SVJ2F model.

To summarize, the assumption of jumps is crucial for the long-run risk models to generate large variance risk premiums. The jump-in-long-run-growth models are generally more effective at generating large variance risk premiums than jump-in-volatility models. However, the cost of assuming jumps in long-run growth is the reduced persistence in realized volatility. Finally, including a second volatility factor enables the model to display long-range dependence in the risk-neutral expected volatility.

3.2 Consumption and Dividend

Although we do not directly include the data regarding consumption and dividend growth in our calibration, it is interesting to compare the implied consumption and dividend of the LRR models and the annualized data. Table 8 reports such a comparison.

For the annual consumption, all models match the mean of the annual consumption growth very well. Yet they all generate a slightly higher volatile consumption process than the sample counterpart. The models also imply that the distribution of consumption is more normal than the sample. The medians of skewness are all very small and the medians of the kurtosis are close to 3, in contrast to the negative sample skewness of -0.59 and the high kurtosis of 3.47. However, the sample values are still included in the 90% confidence interval of all models, as the unconditional skew and kurtosis is hard to estimate robustly. As for the first order autocorrelation, the SVJ2F model implies a more persistent consumption process, while all the other models displays mild first order persistence and the value is close to the sample.

For the annual dividend, all the models display a relatively higher dividend growth rate than the data, partly because we set a high unconditional dividend growth rate. All the models imply lower dividend volatility than the sample data. This is because in our calibration, we find that increasing the dividend volatility parameter φ_d typically makes the fit for asset pricing moments worse even though it brings the dividend volatility closer to the sample. The sample dividend growth displays a small positive skewness and large kurtosis. They are generally within the 90% confidence intervals generated by the models. Finally, the autocorrelation of the dividend growth is matched by the models reasonably well.

[insert Table 8 about here]

3.3 Robustness Check

Since our models have many preset parameters, it is important to check whether changing these parameters would produce significantly different results.

- Discount factor and dividend dynamics

In our calibration, we set the discount factor at 0.999 and chosse the dividend dynamics parameters close to Bansal et al. (2007a). In our study, we have tried four different subjective discount factors ($\delta = 0.999, 0.9985, 0.998, 0.997$) and find that only the discount factor of 0.999 allows the model to fit interest rates and price-dividend ratios simultaneously well. We have also tested several values of ϕ_d, (the dividend growth's loading factor on x_t). At least from 2.5 to 3.5, we find that similar sample moments can be achieved by small adjustments of other parameters. Parameter φ_d directly control the volatility of dividend process. As we have mentioned, during our calibration process, we have varied φ_d from 5 to 11 to check their impacts on the model fit. We find that as φ_d gets larger than 8, the fit for the model typically gets worse, but the dividend volatility is closer to the sample data. So whether we can fit both the moments of the dividend process and the moments of asset prices is still an open question. Finally, we test how changing the coefficient ρ_{dc} would impact the model results. We choose a typical value of 0.4, but varying it from 0.2 to 0.6 does not materially impact the quality of the model fit, as other parameters can be adjusted in response to the change in ρ_{dc}.

- Leverage-effect coefficient

In our model, we restrict the shock correlation coefficients ρ_{xf} to -0.8. We find that this coefficient has little effect on monthly aggregate asset moments; however, setting ρ_{xf} to -0.8 can potentially significantly improve the model's description of daily dynamics dependence between stock returns and the VIX index.

- Jump intensity and jump size

We also test whether the calibrated jump parameters will affect the result of estimation. In the SVJ2F, SVJ1F_X, and SVJ1F_X_SM models, the parameters of the jump intensity coefficients l_{XV} and l_V are both set to be 4000. While in the SVJ1F_V and SVJ1F_V_SM the coefficient of l_V is set to be 500. It is extremely hard to identify jump intensity and jump size jointly. Our experiments suggest that if jump intensity is only varied by order of 50%, then the impact on sample moments can be largely offset by changes in jump size parameters (i.e. lower jump intensity requires higher jump skewness).

We also assume that the jump size for the long-run risk factor is negatively Gamma distributed. We experiment with setting the jump as exponentialy distributed to test whether particular jump size distribution is affecting the result. Consistent with Drechsler et al. (2009), the difference between the exponentialy distributed jump and Gamma distributed jump mainly lies in their effectiveness in generating the variance risk premium. It is generally more difficult for an exponentialy distributed jump in the long-run component to generate a large variance risk premium.

- A different volatility setup

In the SVJ2F model, we assume that the volatility of short-run consumption growth is controlled by the persistent volatility factor, while the long-run consumption growth is controlled by the non-persistent volatility factor. Here we test a slightly different volatility setup, in which the persistent volatility factor V_t^p controls the diffusion term of the long-run risk factor, while the fast mean-reverting factor V_t^f controls the diffusion term of the short-run consumption and dividend. This specification is similar to Bansal et al. (2010). However, we find that the model cannot achieve a reasonable fit to to the data moments, therefore, we believe it is unlikely to be a realistic specification.

3.4 Predictability of Growth Rate,Return, and their Volatility

Bansal and Yaron(2004) suggest that the LRR model can generate the strong predictability of excess stock returns by price-dividend ratios. The conclusion is further supported by Drechsler and Yaron (2009). However, Beeler and Campbell (2009) raise doubts about the reliability of this result and other counterfactual implications of the LRR model. Since the work of Beeler and Campbell (2009) is based on the canonical model of Bansal and Yaron (2004), whether the conclusion holds for more general models remains in question. Here using our newly calibrated model we reinvestigate this issue and show that the conclusions reached by Beeler et al. (2009) are largely intact.

The predictive regression for H periods future compound excess stock returns is:

$$\sum_{i=0}^{H-1}(\ln R_{t+i,t+i+1} - \ln R_{f,t+i}) = \alpha + \beta_{pred}(p-d)_t + \epsilon_{Ht}, H \geq 1 \tag{32}$$

where $R_{t+i,t+i+1}$ represents the market return from the period $t+i$ to the period of $t+i+1$. Similarly, the predictive regressions for log consumption and log dividend growth rates are:

$$\begin{aligned} \sum_{i=0}^{H-1}(\triangle c_{t+i,t+i+1}) &= \alpha_c + \beta_{pred\_c}(p-d)_t + \epsilon_{Hct} \\ \sum_{i=0}^{H-1}(\triangle d_{t+i,t+i+1}) &= \alpha_d + \beta_{pred\_d}(p-d)_t + \epsilon_{Hdt}. \end{aligned} \tag{33}$$

While the regressions of consumption and dividend growth are based on quarterly data; the regression for excess stock returns is based on the monthly bivariate Vector Autoregressive Regression (VAR) model with lag order of 1. Hodrick (1992) shows that the VAR model can reduce the bias brought by finite sample and overlapping returns.

Furthermore, we examine whether the model and data would agree on the predictability of stock realized volatility, consumption volatility, and dividend volatility by price-dividend ratios. The return volatility is measured as the log of the aggregate realized volatility over the predictive horizon, specifically, the dependent variable is defined as

$$Vol_{t+1,t+H} = \frac{1}{2}\ln \Sigma_{h=0}^{H}|RV_{t+h-1,t+h}|,$$

where $RV_{t-1,t} = [\Sigma_{k=0}^{K-1}(p_{t-1+\frac{k+1}{K}} - p_{t-1+\frac{k}{K}})^2]$ is the realized variance in month t.

The consumption and dividend volatilities are measured based on the non-parametric method proposed in Bansal et al. (2005). Specifically, for each variable y_t (which can be quarterly consumption or dividend), AR(1) regressions of consumption and dividend are run and the absolute values of the residuals ϵ_{ct} and ϵ_{dt} are used to characterize the realized volatilities of the consumption and dividend respectively. So the H-quarter realized volatility of consumption or dividend is defined as the sum of the quarterly realized volatility:$Vol_{t+1,t+H} = \Sigma_{h=1}^{H}|\varepsilon_{y,t+h}|, y \in (c, d)$. Then the volatility predictive regression is:

$$\ln[Vol_{t+1,t+H}] = \beta_{v0} + \beta_{v1}(p-d)_t + \xi_{Ht}. \tag{34}$$

Table 9 Panel A reports the regression coefficients, t-ratios, and R^2s of the regressions of returns and growth rates at horizons of four, twelve, and twenty quarters. As shown in the three columns at the left, the regression results based on the data are largely consistent with what is found in the literature, i.e. price-divided ratios strongly predict stock returns but have weak or no predictability on consumption and dividend growth rates.

Right to the empirical regression results are the regression results based on the simulated data from all the models including jumps. For the predictability of excess returns, we find that the R^2 of the regression steadily increases from left to right. This trend suggests that model-implied price-dividend ratios' predictive power gets stronger when the multiple volatility factors are included. However, even the largest R^2 generated by the model is still significantly smaller than what is found in the empirical study. The R^2 in our model is also consistently smaller than the model-implied R^2 in other studies such as Drechsler et al. (2009)(see Table IX, median value). As suggested in Drechsler et al. (2009), the model implies a large finite sample fluctuation in the value of R^2. Another important reason is that compared to Drechsler et al. (2009), we put more constraints on the persistence structure of the market volatility.

The model does a poor job in matching the predictive regression results for consumption/dividend growth. Specifically, all the models imply that price-dividend ratios strongly predict the growth of consumption and dividend, especially at long horizons. Our results essentially replicate the critiques raised by Beeler and Campbell (2009). However, we note that the results here need to be interpreted with caution. Although it could be true as Beeler and Campbell (2009) suggest that the long-run component indeed does not exist in consumption and dividend, it may be equally true that the consumption and dividend used to test these models are seriously mismeasured. Currently consumption is measured as the sum of service and non-durable goods, yet durable goods can also play an important role in asset pricing (e.g. Yang 2009). Furthermore, the consumption of the stock participants may be a better measure of consumption for pricing assets (see Vissing-Jorgenson 2002)[11]. Using dividends to represent

[11] Several other theories are also able to explain the mismatch regarding the predictability of consumption growth. Bonomo et al. (2009) suggest that using Generalized Disappointment Aversion to characterize investors

corporate cash flows might also be subject to biases, as dividends can be affected by shifts in corporate financial policy. Several studies reveal important connections between asset prices and earnings (see Longstaff and Piazzesi 2004, Bansal et al. 2005). In fact, although our model is not calibrated based on earnings of the market, the SVJ1F_V_SM model implies that the R^2 of one year horizon of the price-dividend ratios' predictictive power is 0.07, quite close to the predictability of earning growth by price-earning ratios from 1949 to 1999, as shown in Table 6 of Bansal et al. (2005). Therefore, it is quite possible that earnings also play an important role for investors to value assets.

When comparing the regression results between the data and the model regarding the predictability of the volatility of growth rates and returns by price-dividend ratios, as reported in Panel B of Table 9, we again find large mismatches. In the empirical side, we find that the price-dividend ratio with dividend adjusted by stock repurchases cannot predict volatility of consumption, dividend and stock returns. On the other hand, all the models suggest that price-dividend ratios can strongly predict all volatilities. How to reconcile this discrepancy is a largely open question.

3.5 Unconditional Decomposition of Equity Risk Premium

Table 10 reports the results for decomposing the instantaneous expected equity risk premium into parts that are attributed to different risk resources. The first row reports the unconditional mean of the equity risk premiums for all models. The second to fifth rows record the premiums attributed to diffusion risks in (short-run) consumption, long-run risk factor, short-run volatility factor, and long-run volatility factor. The sixth and seventh rows record the equity premiums contributed by jumps in the long-run factor and the volatility of the long-run factor.

For the SV1F and SV2F models, the diffusion risk associated with the LRR component x_t constitutes most of the equity risk premium commanded by investors, both close to 90%. When jumps are included, the risk premium attributed long-run risk is significantly reduced, except for the SVJ1F_V model the percentage is typically not more than 50%. Investors command significant equity premiums to compensate risks in jumps. For jump-in-volatility models, the jump risks contribute to about 20% of the equity risk premium. In jump-in-long-run-factor models, the percentage reaches as high as 46% (for the SVJ2F model).

[insert Table 10 about here]

Table 10 reveals quantitatively how much risk premiums can jump risks account for. As almost all of the variance risk premium is due to the jump. This indirectly suggests that

utility function can explain the low predictability in the consumption growth implied by the LRR model. Hansen and Sargent (2009) propose that model uncertainty for investors can lead investors to choose to believe that a long-run risk component exists in consumption dynamics.

variance risk premiums are closely associated with equity premiums. Our result is consistent with what Bollerslev et al. (2010) find. Using non-parametric methods, they find that the risk aversion associated with the tail events not only generates the large variance risk premium, but also constitutes more than 60% of the equity risk premium. Furthermore, our conclusion is consistent with both theoretical and empirical studies that suggest that the risk premium in the option market illustrates the importance of compensation for risks associated with rare events. Barro (2006) documents an internationally nontrivial probability of rare disasters that can cause a crash in consumption growth. Gabaix (2008) further extends the model and shows that the rare disaster model can generate a high implied volatility skew, which is closely related to variance risk premium. In our LRR models, the infrequent large negative jump in the long-run growth rate/volatility indeed shares certain characteristics of a rare disaster, however, in the LRR model the immediate impact on the economy is much smaller. Since this drop occurs in the expected consumption growth rate instead of in consumption itself, the model can generate a large price drop when consumption has relatively little change, which is at least consistent with what we have observed in the U.S. economy.

3.6 Variance Risk Premium's Predictability of Stock Returns

We now turn to the predictability of stock excess returns by the variance risk premium. Both Bollerslev et al. (2009) and Drechsler et al. (2009) find that the variance risk premiums can forecast future stock returns at short to medium horizons.[12] The sample periods in these studies typically end in 2007, just before the financial crisis. In addition to this period, we also investigate the period that includes the financial crisis and post-crisis periods ending in July 2010. To calculate the variance risk premium, we estimate the expected realized variance based on the HAR-RV method.[13] The univariate regression is

$$\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t}) = \beta_0 + \beta_1 VRP_t + \varepsilon_t, \tag{35}$$

where J represents the forecasting horizon. Here we obtain the regression statistics implied by the VAR forecasting model for predicting compound returns at the horizon of one, three, six, nine, and twelve months. The VAR analysis has been proved to be able to reduce biases caused by finite sample and overlapping returns. To compare, in the empirical analysis part we also add the results based on OLS regression with t-ratios adjusted as Hodrick (1992). The standard variance risk premium is measured based on HAR-RV method; to compare, we also add the

[12] They define the return for horizon h months as the mean returns from month $t+1$ to month $t+h$

[13] In Bollerslev et al. (2009), the measure of the variance risk premium is the difference of VIX^2 and the lagged realized variance. In Drechsler et.al. (2008), the expected variance is estimated based on running a regression of realized variance on the past month's realized variance and the VIX^2.

case when the variance risk premium is measured based on the difference between expected model-free implied variance and lagged realized variance, as in Bollerslev et al. (2009).

At the model side, we only use the VAR based analysis as OLS regression gives qualitatively similar result for long simulation. However, we use various methods to measure the variance risk premium in the simulated data, aside from using the true expected physical expected variance, we also use the HAR-RV method and Bollersleve et al. (2009)'s method to estimate the expected variance.

In both empirical and model-implied data, we compare the predictability of excess returns based on the model-free implied variance, measured as the VIX^2.

[insert Table 11 about here]

Table 11 reports the empirical regression results. The main message reflected in this table is that the predictability of excess returns by the variance risk premium is sensitive the estimation period, the way the variance premium is measured, and the way the regression analysis is conducted. The upper panel of Panel A reports the regression coefficients and the R^2 implied by the monthly VAR analysis, when the physical expected variance is estimated by the HAR-RV method. From 1990 to 2007, the variance risk premium indeed has a short-run predictability on variance risk premium, with the strongest predicting power at a horizon of three months. For the 1990-2010 period, however, the predictability largely disappears. In the bottom panel of Panel A, the variance risk premium is based on using the lagged realized variance as the physical expected variance, and it has no predictability on excess returns from 1990 to 2007 and weak predictability from 1990 to 2010.

In Panel B, we conduct the OLS regression to investigate the predictability of excess returns by the variance risk premium. We find that when the variance risk premium is constructed based on the HAR-RV method, the predictability pattern is similar to that in Panel A, i.e. there is statistically significant predictability from 1990 to 2007 and no predictability from 1990 to 2010. When the variance risk premium is estimated based on lagged realized variance, there is significant predictability in both periods.

From the comparison, we can see that both the financial crisis and the short-time span make inference significantly difficult, however, we believe that the HAR-RV method is a more appropriate method to estimate the expected variance, VAR method is more robust against finite sample, and the period from 1990 to 2007 represents a more normal market condition. Therefore, our conclusion is that the variance risk premium can predict excess returns at short horizon. Finally, Panel C illustrates that VIX^2 cannot predict excess returns in both periods, which is consistent with most previous studies such as Bollerslev et al. (2009).

[insert Table 12 about here]

Table 12 reports the predictability test based on long period simulations from different LRR models. The upper panel reports the regression result when the physical expected variance is constructed by the "true" expected variance deduced from the model. The second part reports the regression results when the physical expected variance is estimated using the HAR-RV method. The third part reports the regression results when the lagged realized variance is used as the physical expected variance. The bottom part is the regression results when VIX^2 is used as the regressor variable.

For the first measure of the variance risk premium, all regressions suggest that the variance risk premium can strongly predict excess stock returns. For the second measure, the predictability power generally decreases, especially for long horizon forecasting. Yet the SVJ1F_V_SM model generates regression coefficients and R^{2}s that are quite close to the empirical regression results in the upper left part of Pane A in Table 11. For the third measure, where the lagged realized variance is used, all models imply large R^{2}s at short horizons but R^2 decreases fast as horizon increases. The t-ratios, however, are significant at all horizons. This pattern clearly is different with what is observed in the data.

Although the SVJ1F_V_SM model can match the predictive pattern of excess returns, it still has serious limit. As shown in the bottom panel, all models, including the SVJ1F_V_SM model imply that the expected risk-neutral variance can strongly predict excess returns, which is in sharp contrast with the data. This is not surprising to some extent, as in all jump-based long run risks models, the conditional variance in risk-neutral measure or physical measure and expected stock returns are linear functions of the multiple volatility factors V_t^f and V_t^p. Therefore, the variance risk premium is also a linear function of these two volatility factors. When the variance risk premium predict excess returns, so do the expected variance under the risk-neutral measure. To fully explain the predictability of returns by the variance risk premium but no predictability of returns by VIX^2, we probably need to relax the implicit assumption that conditional variance and equity premium both have constant loadings on volatility factors.

4 Concluding Remarks

In this study, we develop several alternative continuous-time long-run risks models and investigate the relationship between the equity risk premium and the variance risk premium. The central building blocks of the most successful models are the assumptions of jumps and multiple volatility factors. When jumps are assumed, the model can qualitatively generate many key features in asset markets, especially the large variance risk premium coexisting with the high equity risk premium in the market. The assumption of the second volatility factor enables the model to capture the long-range dependence in stock return volatility and stronger predictability of stock returns by key variables such as price-dividend ratios and the variance risk premium in the variance swap markets. Although models with jumps in long-run risk factors generate variance risk premium more effectively than models with jumps in volatility, they generate a persistence in realized volatility much smaller than that found in the data.

Although the LRR model achieves a reasonably good fit in the unconditional moments of both the fundamental data and asset pricing data, there are still several discrepancies between model and data that need to be explained. Firstly, the data suggests no strong predictability of consumption and dividend by price-dividend ratios, in contrast to what models implied. Secondly, the data suggests no predictability of volatility of returns, consumption and dividend, but all models suggest strong predictability. Thirdly, the data imply that the variance risk premium can strongly predict excess returns while the expected variance cannot; but all models suggest that both the variance risk premium and the expected variance can strongly predict excess returns. Despite these drawbacks, we think the long-run risks models do an admirable job in describing the market dynamics.

There are many interesting questions that remain to be answered. In the current models, we still assume that investors have constant utility functions and their sensitivity to volatility factors is constant over time. However, it is well-known that during crisis times investors might be more sensitive to uncertainties in the economy. Therefore, it might be desirable to establish a LRR model in which investors display a time-varying or state-dependent risk aversion.

Additionally, the current model assumes that "uncertainty in the long-run consumption growth" is the main driver of short-term volatility. However, as we all know, there are many other factors that contribute to short term volatility, such as liquidity and inflation uncertainties. These short-term volatilities may well be an integral part in explaining the large magnitude of the variance risk premium. The model established by Calvet and Fisher (2008) offering a promising solution to this problem.

Finally, though the LRR model has been quite powerful in explaining asset dynamics, we are aware that there is a lack of direct evidence for such a long-run component in aggregate consumption. Several theories are proposed to solve this puzzle. The first theory suggests that the measurement of consumption that is more relevant to pricing asset should either include durable goods or account for the limited stock market participation effect (Vissing-Jorgenson 2002Vissing-Jørgensen (2002)). The second theory suggests that the long-run risk assumption may originate from the model uncertainty of investors (Hansen et al.2010). The third theory suggests that the low predictability in consumption growth can be explained by generalized disappointment aversion (Bonomo et al. 2010). We expect to continue our investigations and gain more insight into which of these explanations is correct.

Appendix

A. Solving the Equilibrium Model

A.1 Fundamental and Pricing Kernel Dynamics

The vector $Y_t = [\log C_t, x_t, V_t^f, V_t^p, \log D_t]'$ includes all the variables of interest in our study. Following Eraker and Shaliastovich (2008), the dynamics of Y_t is:

$$dY_t = \mu(Y_t)dt + \Sigma(Y_t)dW_t + \xi_t \cdot dN_t \qquad \text{(A.1)}$$

where

$$\mu(Y_t) = M + KY_t$$

$$\Sigma(Y_t)\Sigma(Y_t)' = h + \sum H_i Y_t.$$

Here M is a $n \times 1$ vector, K is a $n \times n$ matrix, h is a $n \times n$ matrix, and $H_{i \in \{1,\ldots n\}}$ are $n \times n$ matrices. dW_t is a Brownian motion vector, $dN_t = [0, dN_x, dN_V, 0, 0, 0]$ represents the jump intensity state variables. They are governed by Poisson processes, with $Prob(dN_x = 1|\digamma_t) = l_{XV} V_t^f dt$ and $Prob(dN_V = 1|\digamma_t) = l_V V_t^f dt$. $\xi_t = [0, \xi_x, \xi_V, 0, 0]$ represents the jump size. The jump size in x_t follows a compensated negative Gamma distribution, i.e.

$$\xi_x \sim -\Gamma(\mu_x, \gamma_x) + \mu_x$$

The jump size in V_t^f follows a standard Gamma distribution:

$$\xi_V \sim \Gamma(\mu_V, \gamma_V)$$

We assume the representative agent has a recursive utility function as follows:

$$U_t = \{(1 - e^{-(\ln\delta)dt})C_t^{1-\rho} + (e^{-(\ln\delta)dt})E_t(U(t+dt)^{1-\gamma})^{\frac{1-\rho}{1-\gamma}}\}^{\frac{1}{1-\rho}}$$

where $\rho = \frac{1}{\psi}$.

We get the formula describing the dynamics of $d\log M_t$ as:

$$d\log M_t + (1-\theta)dr_t + \frac{\theta}{\psi} d\log C_t = (\log\delta)\theta dt \qquad \text{(A.3)}$$

which is similar to the evolution of the pricing kernel under discrete time

$$\log M_{t+1} = \log(\delta)\theta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1)r_{a,t+1}$$

A.2 Solving the Model

Under the equilibrium, for the consumption-claim asset (wealth), we have

$$\begin{aligned} d(\log(M_t R_{c,t})) &= \theta(d\log(P/C)_t + d\log C_t + \frac{dt}{(P/C)_t}) \\ &\quad -\frac{\theta}{\psi} d\log C_t + \log(\delta)\theta dt \\ &= 0 \end{aligned} \tag{A.4}$$

For the dividend-claim based asset (equity), we have

$$\begin{aligned} d(\log(M_t R_{d,t})) &= \theta(d\log(P/D)_t + d\log D_t + \frac{dt}{(P/D)_t}) \\ &\quad -\frac{\theta}{\psi} d\log D_t + (\log\delta)\theta dt \\ &= 0 \end{aligned} \tag{A.5}$$

Under the log-linear approximation, we have

$$dr_{c,t} = k_0 dt + k_1 dv_t^c - (1-k_1)v_t^c dt + d\log C_t$$

$$dr_{d,t} = k_{0d} dt + k_{1d} dv_t^d - (1-k_{1d})v_t^d dt + d\log D_t$$

Assuming $v_t^c = A + BY_t$, $v_t^d = A_d + B_d Y_t$. Substituting into the Euler equation A.4 and A.5 above, we get

$$K'\chi - \theta(1-k_1)B + 0.5\chi' H\chi + l_1^{'}(\varrho(\chi) - 1) = 0 \tag{A.5}$$

$$\theta(\ln\delta + k_0 - (1-k_1)A) + M'\chi = 0 \tag{A.6}$$

$$K'\chi_d + (\theta-1)(k_1-1)B + (k_{1d}-1)B_d + 0.5\chi_d' H\chi_d + l_1^{'}(\varrho(\chi_d) - 1) = 0 \tag{A.7}$$

$$\begin{aligned} &\theta(\ln\delta - (\theta-1)(\ln k_1 + (k_1-1)B'\mu_Y) - (\ln k_{1d} + (k_{1d}-1)B_d'\mu_Y \\ &+M'\chi_d + 0.5\chi_d' h\chi_d + l_0[\rho(\chi_d) - 1] = 0 \end{aligned} \tag{A.8}$$

$$A_d + B_d'\mu_Y = \ln\frac{\kappa_{1,d}}{1-\kappa_{1,d}}. \tag{A.9}$$

Here $\chi = \theta((1-\rho)\delta_c + k_1 B), \chi_d = \delta_d + k_{1d}B_d - (\gamma\delta_c + (1-\theta)k_1 B), \mu_Y$ is the unconditional mean of state variables (for $\log C_t$, $\log D_t$, the unconditional mean is set to zero).

The coefficients k_0, k_1, k_{0d}, k_{1d} are not known initially, but we can assume the value and recursively solve them using the relation:

$$k_1 = \frac{\exp(E(v_t^c))}{1+\exp(E(v_t^c))} \tag{A.10}$$

$$k_0 = -\ln[(1-k_1)^{1-k_1} k_1^{k_1}]$$

and

$$k_{1d} = \frac{\exp(E(v_t^d))}{1+\exp(E(v_t^d))}$$

$$k_{0d} = -\ln[(1-k_1)^{1-k_1} k_1^{k_1}]$$

A.3 Risk-free Rate

Using the fact that $M_t e^{\int_0^t r(s)ds}$ is a martingale, with Ito's lemma, we can derive the instantaneous risk-free interest rate as

$$r_t = \Phi_0 + \Phi_1 Y_t \tag{A.11}$$

here

$$\Phi_0 = \theta\beta + (\theta-1)(\ln k_1 + (k_1-1)B'\mu_{Y_t}) + M'\lambda - \frac{1}{2}\lambda' h\lambda,$$

and

$$\Phi_1 = (1-\theta)(k_1-1)B + K'\lambda - \frac{1}{2}\lambda' H\lambda - l_1'(\varrho(-\lambda)-1)$$

where $\lambda = \gamma\delta_c + (1-\theta)k_1 B$ is the price of risks. The calculation of Φ_0 and Φ_1 follows Eraker (2008)

A.4 Risk Neutral Dynamics

The risk neutral dynamics of state variables can be expressed as

$$dY_t = (M^Q + K^Q Y_t)dt + \Sigma(Y_t)dW_t^Q + \xi_t^Q \cdot dN_t^Q \tag{A.12}$$

where

$$M^Q = M - h\lambda$$

$$K^Q = K - H\lambda$$

$$dW_t^Q = dW_t + \Lambda_t dt,$$

where $\lambda = \gamma\delta_c + (1-\theta)k_1 B$. λ reflects how different shocks of state variables can affect the change in the stochastic discount factor. $\Lambda_t = \Sigma(X_t)'\lambda$ can be understood as the price of the diffusion shock.

The jump arrival intensity transformation under risk neutral measure follows

$$l_{xV}^Q = \exp(-\lambda_x\mu_x)(1-\frac{\lambda_x\mu_x}{\gamma_x})^{-\gamma_x} l_{xV}, \tag{A.13}$$

$$l_V^Q = (1 + \frac{\lambda_v \mu_v}{\gamma_v})^{-\gamma_v} l_V$$

For jump size distribution, we get the risk neutral moment generating function as

$$\varrho_x^Q(u) = \exp(\mu_x u)(1 + \frac{\mu_x^Q u}{\gamma_x^Q})^{-\gamma_x^Q}$$

$$\varrho_V^Q(u) = (1 - \frac{\mu_V^Q u}{\gamma_V^Q})^{-\gamma_V^Q}$$

here $\gamma_x^Q = \gamma_x, \mu_x^Q = \frac{\mu_x \gamma_x}{\gamma_x + \lambda_x \mu_x}, \gamma_V^Q = \gamma_V, \mu_V^Q = \frac{\mu_V \gamma_V}{\gamma_V + \lambda_V \mu_V}$

A.5 Calculation of the Risk Neutral Expected Variance (VIX^2)

For a large enough n, we can calculate VIX^2 as:

$$\begin{aligned} VIX^2 &= E_t^Q[\sum_{j=1}^n Var^Q_{t+\frac{j-1}{n}}(p_{t+\frac{j}{n}} - p_{t+\frac{j-1}{n}})] \\ &= \sum_{j=1}^n E_t^Q[Var^Q_{t+\frac{j-1}{n}}(p_{t+\frac{j}{n}} - p_{t+\frac{j-1}{n}})] \\ &= \sum_{j=1}^n E_t^Q[\alpha''(0, \frac{1}{n}) + \beta''(0, \frac{1}{n}) X_{t+\frac{j-1}{n}}] \\ &= \alpha''_Q(0, \frac{1}{n}) + \beta''_Q(0, \frac{1}{n}) \sum_{j=1}^n E_t^Q(X_{t+\frac{j-1}{n}}) \end{aligned} \quad \text{(A.14)}$$

Here $\alpha_Q(u,t)$ and $\beta_Q(u,t)$ satisfies

$$\dot{\beta}_Q = K^{Q'}\beta + \frac{1}{2}\beta' H \beta + l_1^{Q'}(\varrho^Q(\beta) - 1)$$

$$\dot{\alpha}_Q = M^{Q'}\beta + \frac{1}{2}\beta' h \beta$$

with initial condition $\alpha(u,0) = 0$ and $\beta(u,0) = u$.

As mentioned above, in our model, we find that $n = 1$ can provide a good approximation of the risk neutral expectation of the model.

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Table 1: Different specifications studied in this paper

	SV1F	SV2F	SVJ1F_X	SVJ1F_V	SVJ1F_X_SM	SVJ1F_V_SM	SVJ2F
δ_{c1}	1	0	1	1	1	1	0
a_{f1}	0	1	0	0	1	1	1
l_{XV}	0	0	>0	0	>0	0	>0
l_V	0	0	0	>0	0	>0	>0

Table 2: Description of Data Sets

Cash Flow:
Annual Consumption: non-durable and service real consumption per capita, 1951-2010
Annual Cash Dividend: Dividend deduced from CRSP value-weighted market returns with without dividend
Annual Adjusted Dividend: Cash dividend plus payout to shareholders in the form of share repurchases, using formula in Boudoukh (2007)
Asset Prices (from January 1951 to July 2010)
Monthly Real interest Rates: Nominal interest rate (measured by 3 month Treasury Bill) adjusted by forecasted inflation rate (before 1967, measured by 3 month moving average of ex post inflation, since 1967, measured by GDP deflator obtained from Survey of Professional Forecasters)
log price-dividend ratios: log of the ratio of monthly real S&P 500 index price and the sum of the dividend paid over the past 12 months.
Monthly Market Excess Returns: the difference between market returns (measured by the sum of S&P index returns and cash dividend yield) and nominal risk free rate (measured by 3-month treasury bond), adjusted by ex post inflation.
Monthly aggregate realized variance: before 1990, measured by aggregated daily squared return; after 1990, measured by aggregate realized variance based on 5-min returns for S&P 500 futures.
Aggregate Variance Data January 1990- July 2010
Monthly Aggregate Realized Variance(RV_{post}): aggregate realized variance based on 5-min S&P 500 futures.
Monthly option-implied variance (VIX^2): squared VIX reported by CBOE at the end of each month.
Monthly Variance Risk Premium based on HAR-RV(VRP_{HAR}): the difference between monthly option-implied variance and physical expected variance (measured by HAR-RV method)

Table 3: Summary Statistics

This table presents the summary statistics of the data used in estimation and further analysis. The numbers in parenthesis indicates the sample bootstrap confidence interval. All variables are reported in annualized percentage form whenever appropriate except that RV, VIX^2, and VRP_{HAR} are recorded in units of 1/12 of annual variance.

Annual: 1951-2010					
	Mean	Std.dev.	Skewness	Kurtosis	AC(1)
Δc	2.03	1.22	-0.59	3.47	0.38
	[1.71,2.35]	[0.96,1.46]	[-1.11,0.04]	[1.86,4.75]	[0.13,0.63]
Δd_{cash}	1.00	6.90	0.42	4.85	0.18
	[-0.79,2.81]	[5.12,8.54]	[-0.71,1.38]	[2.45,6.60]	[-0.07,0.44]
Δd_{payout}	2.52	12.52	0.18	5.51	0.35
	[-0.51,5.62]	[8.87,15.79]	[-1.01,1.50]	[2.95,7.43]	[0.10,0.61]
Monthly: 1951m1 to 2010m7					
	Mean	Std.dev.	Skewness	Kurtosis	AC(1)
R_f	1.68	2.03	0.06	3.49	0.88
	[1.53,1.82]	[1.90,2.15]	[-0.20,0.31]	[2.96,3.98]	[0.80,0.95]
log(P/D)	3.17	0.29	-0.24	2.04	0.98
	[3.15,3.20]	[0.28,0.30]	[-0.37,-0.13]	[1.89,2.20]	[0.91,1.00]
R_m-R_f	6.70	14.76	-0.40	4.68	0.05
	[2.97,10.39]	[13.73,15.72]	[-0.78,-0.01]	[3.13,6.09]	[0.02,0.12]
RV	19.78	39.87	11.32	166.21	0.40
	[16.89,22.66]	[20.45,57.15]	[5.84,14.86]	[38.58,263.05]	[0.33,0.47]
Monthly: 1990m1-2010m7					
	Mean	Std.dev.	Skewness	Kurtosis	AC(1)
RV_{post}	29.71	49.36	7.98	87.2	0.55
	[23.72,35.45]	[19.04,73.95]	[1.83,10.82]	[0,129.4]	[0.42,0.67]
VIX^2	39.75	36.47	3.33	18.92	0.82
	[35.23,44.21]	[26.52,45.26]	[1.98,4.23]	[6.74,27.2]	[0.69,0.94]
VRP_{HAR}	13.87	23.70	4.37	35.68	0.38
	[10.97,16.67]	[14.91,31.33]	[1.04,6.33]	[0.78,54.7]	[0.25,0.50]

Table 4: Adjusted Sample Moments

This table presents the adjusted moments used in calibration.

	Mean	Std	AC(1)	AC(6)	AC(12)
R_f	1.51	1.61	0.95	-	-
log(P/D)	3.19	0.27	-	-	-
RVol	13.86	7.91	-	-	-
VIX	18.11	7.67	0.86	0.58	0.48

Table 5: Calibration and Estimation of the Long-run Risks Model

This table reports calibration and estimates of various long-run risks models. Panel A reports the parameters that are calibrated. Panel B reports the estimation result. The optimal weighting matrix is calculated based on the Newey-West method with Bartlett kernel lag length of 10. The table also reports the overidentifing J-statistics along with the associated p-value. All the parameter units are at monthly frequencies.

Panel A: Preset Parameters

Parameter	SV1F	SV2F	SVJ1F_X	SVJ1F_V	SVJ1F_X_SM	SVJ1F_V_SM	SVJ2F
δ	0.999	0.999	0.999	0.999	0.999	0.999	0.999
μ_c	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017
μ_d	0.002	0.002	0.0028	0.0028	0.0028	0.0028	0.0028
ϕ_d	3	3	3	3	3	3	3
ρ_{dc}	0.4	0.4	0.4	0.4	0.4	0.4	0.4
ρ_{xf}	-0.8	-0.8	-0.8	-0.8	-0.8	-0.8	-0.8
l_{XV}	0	0	4000	0	4000	0	4000
γ_x	1	1	1	1	1	1	1
l_V	0	0	0	500	0	500	4000
γ_V	1	1	1	1	1	1	1

Panel B: Systematically calibrated parameters

Parameter	SV1F	SV2F	SVJ1F_X	SVJ1F_V	SVJ1F_X_SM	SVJ1F_V_SM	SVJ2F
φ_d	6.5	6.5	6.5	6.5	8	6.5	5
γ	9.32	7.15	7.40	7.24	8.06	7.98	6.01
ψ	1.54	1.71	1.65	1.37	1.75	1.42	1.55
κ_x	0.003	0.009	0.009	0.005	0.005	0.005	0.016
φ_e	0.016	0.014	0.022	0.022	0.023	0.021	0.053
$\sigma_w^f(\times 10^4)$	32.2	15.28	20.96	2.05	20.06	3.26	37.5
κ_v^f	0.28	1.40	0.30	0.35	0.23	0.18	0.50
$\bar{V}_f(\times 10^5)$	2.18		2.43	2.34	-	-	-
$\bar{V}_p(\times 10^5)$	-	0.15	-	-	0.83	1.80	1.65
$\sigma_w^p(\times 10^4)$	-	3.04	-	-	1.78	2.73	2.52
κ_v^p	-	0.064	-	-	0.018	0.01	0.01
$\mu_x(\times 10^4)$	-	-	3.73	-	3.15	-	8.79
$\mu_V(\times 10^5)$	-	-	-	18.44	-	17.15	2.44
OID	677.1	515.3	133.0	185.7	112.5	139.2	75.1

Table 6: Match in Moments SV Model

This table displays matches in key moments between the estimated models and the monthly U.S. data. Panel A includes the moments that are included in the SMM estimation. Panel B includes the moments that are not included in the estimation. I then report the population statistics estimated SV1F, SV2F, SVJ1F_X, and SVJ2F models. The reported population values are calculated from a simulation of 100,000 months for each model.

	Data	SV1F	SV2F
E(R_f)	1.68	1.47	1.84
		[0.40,2.52]	[1.02,2.54]
σ(R_f)	2.03	1.21	0.93
		[0.98,1.52]	[0.71,1.32]
AC1(R_f)	0.88	0.79	0.90
		[0.72,0.86]	[0.68,0.90]
E(p-d)	3.17	2.85	3.24
		[2.54,3.15]	[2.99,3.45]
σ(p-d)	0.29	0.16	0.22
		[0.10,0.27]	[0.14,0.34]
E(RVol)	13.38	12.72	11.92
		[11.79,13.68]	[10.58,13.65]
AC(RVol)		0.70	0.65
		[0.64,0.74]	[0.58,0.72]
E(VIX)	20.35	13.64	12.62
		[12.25,15.03]	[10.59,15.44]
AC1(VIX)	0.86	0.72	0.78
		[0.63,0.80]	[0.66,0.87]
AC6(VIX)	0.51	0.13	0.48
		[-0.04,0.34]	[0.25,0.67]
AC12(VIX)	0.39	0.00	0.27
		[-0.16,0.18]	[0.01,0.51]
$R_m - R_f$	0.07	0.07	0.05
		[0.05,0.10]	[0.03,0.08]
VRP	13.87	1.98	0.81
		[-0.51,2.44]	[-2.40,3.54]

Table 7: Match in Moments SVJ Model

This table displays matches in key moments between the estimated models and the monthly U.S. data. Panel A includes the moments that are included in the SMM estimation. Panel B includes the moments that are not included in the estimation. we then report the population statistics estimated SV1F, SV2F, SVJ1F_X, and SVJ2F models. The reported population values are calculated from a simulation of 100,000 months for each model.

	Data	SVJ1F_X	SVJ1F_V	SVJ1F_X_SM	SVJ1F_V_SM	SVJ2F
E(R_f)	1.68	1.63	2.07	1.47	2.04	1.89
		[0.81,2.35]	[0.98,3.08]	[0.68,2.11]	[1.16,2.78]	[0.94,2.94]
σ(R_f)	2.03	1.01	1.33	1.08	1.05	1.38
		[0.80,1.38]	[0.98,1.64]	[0.83,1.42]	[0.63,1.57]	[0.97,2.06]
AC1(R_f)	0.88	0.85	0.79	0.83	0.81	0.89
		[0.79,0.92]	[0.72,0.86]	[0.77,0.89]	[0.74,0.91]	[0.81,0.94]
E(p-d)	3.17	3.22	3.21	3.17	3.17	3.22
		[2.99,3.42]	[2.86,3.55]	[2.88,3.41]	[2.86,3.42]	[3.03,3.37]
σ(p-d)	0.29	0.21	0.23	0.20	0.22	0.22
		[0.14,0.32]	[0.16,0.37]	[0.13,0.33]	[0.14,0.35]	[0.15,0.35]
E(RVol)	13.38	14.52	14.67	13.40	13.47	14.37
		[13.56,15.51]	[13.98,14.89]	[11.61,15.41]	[11.34,16.48]	[12.06,17.38]
AC(RVol)	0.63	0.45	0.50	0.62	0.55	0.36
		[0.29,0.58]	[0.44,0.57]	[0.39,0.73]	[0.43,0.71]	[0.17,0.55]
E(VIX)	20.35	18.10	16.88	16.85	16.69	18.59
		[16.61,19.81]	[16.73,17.09]	[13.71,20.74]	[12.84,21.58]	[14.38,24.32]
AC1(VIX)	0.86	0.72	0.75	0.78	0.89	0.70
		[0.62,0.79]	[0.72,0.77]	[0.69,0.86]	[0.79,0.96]	[0.58,0.80]
AC6(VIX)	0.51	0.12	0.21	0.25	0.57	0.19
		[-0.05,0.31]	[0.14,0.27]	[0.03,0.47]	[0.27,0.83]	[-0.01,0.44]
AC12(VIX)	0.39	0.00	0.27	0.09	0.48	0.12
		[-0.17,0.19]	[0.22,0.32]	[-0.13,0.32]	[0.17,0.74]	[-0.07,0.38]
$R_m - R_f$	0.07	0.07	0.06	0.07	0.07	0.07
		[0.04,0.10]	[0.03,0.10]	[0.05,0.10]	[0.04,0.10]	[0.04,0.10]
VRP	13.87	9.80	5.68	8.67	7.91	11.81
		[6.04,14.08]	[5.07,7.83]	[1.91,18.69]	[-0.14,21.38]	[2.13,28.11]

Table 8: Comparison of Consumption and Dividend Growth Dynamics

This table displays the moments and quantile distribution of consumption and dividend growth in the annual U.S data and in the data simulated from the SVJ2F model. For the model I report the population and the finite sample percentile statistics. Population values are calculated from a model simulation of 100,000 months. The percentiles of the statistics are based on 1000 model simulations with each simulation spanning a period of 720 months.

	data	SVJ1F_X	SVJ1F_V	SVJ1F_X_SM	SVJ1F_V_SM	SVJ2F
$E(\Delta c)$	2.03	2.08	2.04	2.05	2.05	2.09
		[0.77,3.16]	[0.53,3.41]	[0.93,3.02]	[0.87,3.20]	[0.69,3.31]
$\sigma(\Delta c)$	1.22	1.73	1.65	1.06	1.35	1.97
		[1.35,2.28]	[1.31,2.17]	[0.79,1.52]	[1.04,1.88]	[1.39,2.92]
$Skew.(\Delta c)$	-0.59	-0.15	0.00	-0.17	-0.01	-0.23
		[-0.75,0.44]	[-0.83,0.74]	[-0.92,0.53]	[-0.82,0.89]	[-1.05,0.50]
$Kurt.(\Delta c)$	3.47	2.90	3.03	3.02	3.15	2.94
		[2.22,4.43]	[2.22,6.04]	[2.18,5.01]	[2.23,6.39]	[2.16,4.70]
$AC1(\Delta c)$	0.38	0.47	0.45	0.53	0.44	0.65
		[0.23,0.69]	[0.19,0.67]	[0.24,0.74]	[0.17, 0.68]	[0.41,0.81]
$E(\triangle d)$	2.52	2.94	2.81	2.94	2.99	3.33
		[-1.39,6.97]	[-2.25,7.62]	[-0.87,6.42]	[-0.93,6.93]	[-0.01,7.00]
$\sigma(\Delta d)$	12.52	9.50	9.26	7.88	7.80	7.47
		[7.87,11.51]	[7.58,11.63]	[6.22,10.05]	[5.97,10.28]	[5.58,10.17]
$Skew.(\Delta d)$	0.18	-0.10	-0.08	-0.15	-0.04	-0.14
		[-0.77,0.51]	[-1.16,-0.96]	[-0.97,0.72]	[-1.19,0.96]	[-0.84,0.55]
$Kurt.(\Delta d)$	5.51	3.10	3.36	3.60	3.43	2.99
		[2.33,4.75]	[2.35,7.43]	[2.51,6.16]	[2.45,7.63]	[2.23,4.48]
$AC1(\Delta d)$	0.35	0.30	0.30	0.27	0.30	0.49
		[0.09,0.50]	[0.06,0.51]	[0.06,0.48]	[0.06,0.50]	[0.24,0.70]

Table 9: Predictability of Excess Returns, Consumption and Dividends

This table presents the coefficients, t-statistics, and R^2s from predictive regressions of excess returns, consumption, and dividend based on log price-dividend ratios.

Panel A. Predictability of return, consumption and dividend growth

	Data			SVJ1F_X1			SVJ1F_V1			SVJ1F_X_SM			SVJ1F_V_SM			SVJ2F		
	$\hat{\beta}_{2010}$	$\hat{t}_{2010}$	$\hat{R}^2_{2010}$	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}
excess return (VAR)																		
4Q	-12.95	-1.99	6.1	-3.13	-1.66	0.3	-3.61	-2.13	0.5	-5.99	-3.40	1.2	-6.79	-4.09	1.8	-10.20	-4.77	2.4
12Q	-32.75	-1.97	14.3	-8.02	-1.65	0.7	-9.46	-2.10	1.1	-15.67	-3.37	3.1	-17.99	-4.05	4.5	-24.68	-4.72	5.5
20Q	-46.33	-1.91	18.7	-11.54	-1.65	0.9	-13.98	-2.08	1.6	-22.97	-3.34	4.3	-26.71	-4.00	6.6	-33.78	-4.64	7.0
consumption growth																		
4Q	0.016	2.03	11.8	0.049	29.35	43.4	0.038	23.31	32.6	0.033	27.24	47.0	0.026	14.87	21.7	0.05	19.32	36.7
12Q	0.028	1.55	8.3	0.133	30.83	56.9	0.105	24.55	50.9	0.092	27.24	59.2	0.070	15.89	33.0	0.12	16.32	33.5
20Q	0.028	1.26	5.2	0.207	29.86	58.2	0.167	26.47	57.8	0.145	27.24	61.1	0.111	16.58	37.5	0.17	14.71	30.5
dividend growth																		
4Q	0.089	1.22	2.5	0.15	13.66	14.9	0.11	10.85	10.4	0.11	10.73	10.5	0.08	6.88	6.9	0.15	12.23	15.8
12Q	0.096	0.53	0.9	0.41	14.63	28.8	0.31	12.18	23.0	0.31	11.86	21.7	0.22	7.53	13.8	0.38	11.29	22.1
20Q	0.066	0.33	0.03	0.63	15.28	35.0	0.50	14.68	31.7	0.49	13.76	29.2	0.36	8.48	18.6	0.58	11.92	23.8

Panel B. Predictability of return, consumption and dividend growth volatility

Periods	Data			SVJ1F_X			SVJ1F_V			SVJ1F_X_SM			SVJ1F_V_SM			SVJ2F		
	$\hat{\beta}_{2010}$	$\hat{t}_{2010}$	$\hat{R}^2_{2010}$	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}	β_{pop}	t_{pop}	R^2_{pop}
realized volatility																		
4Q	-0.18	-0.65	1.9	-0.03	-1.04	0.1	-0.09	-3.22	1.5	-0.33	-7.20	5.8	-0.58	-11.96	24.9	-0.55	-11.66	16.6
12Q	-0.11	-0.44	0.9	-0.05	-1.67	0.6	-0.12	-3.37	2.3	-0.24	-5.64	5.4	-0.56	-10.81	24.4	-0.49	-10.50	18.0
20Q	-0.01	-0.03	0.0	-0.06	-2.23	1.2	-0.13	-3.60	2.9	-0.22	-6.05	6.1	-0.53	-10.16	23.3	-0.48	-10.48	19.7
consumption growth volatility																		
4Q	-0.27	-1.15	2.3	-0.13	-2.46	0.6	-0.05	−0.88	0.0	-0.27	-4.50	2.2	-0.48	-7.14	8.1	-0.50	-9.43	8.5
12Q	-0.33	-1.42	6.8	-0.10	-2.42	1.2	-0.04	-0.96	0.2	-0.19	-3.84	2.8	-0.44	-7.68	13.9	-0.45	-9.81	15.5
20Q	-0.26	-1.41	6.3	-0.10	-2.69	1.7	-0.03	-0.71	0.1	-0.17	-3.94	3.3	-0.41	-7.75	14.7	-0.41	-10.00	16.1
dividend growth volatility																		
4Q	-0.18	-0.50	0.8	-0.13	-2.42	0.5	-0.10	-1.88	0.3	-0.31	-4.87	2.3	-0.58	-8.95	10.9	-0.52	-9.87	8.3
12Q	-0.05	-0.15	0.08	-0.10	-2.51	1.2	-0.09	-2.14	0.9	-0.21	-3.82	2.8	-0.55	-9.33	19.1	-0.48	-9.97	14.6
20Q	0.03	0.10	0.04	-0.10	-2.69	1.7	-0.08	-2.12	1.0	-0.19	-3.96	3.3	-0.51	-9.10	20.1	-0.43	-9.71	15.3

Table 10: Equity Premium Decomposition

This table presents the average contributions to the instantaneous equity risk premium by short-run and long-run consumption growth risks, the long-run growth jump risks, short-run and long-run volatility diffusion risks, and volatility jump risks. The total premium is recorded in annual term.

	SV1F	SV2F	SVJ1F_X	SVJ1F_V	SVJ1F_X_SM	SVJ1F_V_SM	SVJ2F
Total Premium (%)	7.00	5.39	6.56	5.94	6.91	6.31	6.79
$(R_m - R_f\ )_{dW_c}$	9.10%	0.61%	8.47%	8.74%	4.36%	6.95%	4.35%
$(R_m - R_f\ )_{dW_x}$	87.71%	92.05%	46.10%	71.72%	53.24%	52.61%	35.11%
$(R_m - R_f\ )_{dW_{vf}}$	3.20%	0.13%	0.95%	0.00%	2.13%	0.00%	0.28%
$(R_m - R_f\ )_{dWvp}$	-	7.20%	-	-	7.19%	20.42%	13.53%
$(R_m - R_f\ )_{dN_x}$	-	-	44.48%	-	33.08%	-	46.41%
$(R_m - R_f\ )_{dNvf}$	-	-	-	19.53%	-	20.02%	0.33%

Table 11: Empirical Predictability of Stock Returns by Variance Risk Premium

The coefficients, t-statistics, and R^2s are imputed from the monthly VAR regression based on monthly data from 1990 to 2007 and from 1990 to 2010. The returns are defined as the aggregate return with horizon of 1 month, 3 months, 6 months, 9 months and 12 months.

Panel A. VAR regression

Periods	1990-2007			1990-2010		
	$\hat{\beta}_{data}$	$\hat{t}_{data}$	$\hat{R}^2_{data}$(%)	$\hat{\beta}_{data}$	$\hat{t}_{data}$	$\hat{R}^2_{data}$(%)
VRP measure based on HAR-RV (VAR)						
month 1	0.037	1.67	2.3	-0.001	-0.06	0.0
month 1-3	0.064	1.93	2.4	0.005	0.17	0.0
month 1-6	0.069	1.93	1.5	0.005	0.20	0.0
month 1-9	0.069	1.92	1.0	0.006	0.20	0.0
month 1-12	0.069	1.92	0.8	0.006	0.20	0.0
VRP measure based on lagged RV (VAR)						
month 1	0.022	1.28	0.8	0.023	2.33	2.2
month 1-3	0.021	1.29	0.3	0.028	2.20	1.0
month 1-6	0.021	1.29	0.1	0.029	2.17	0.5
month 1-9	0.021	1.33	0.1	0.029	2.17	0.3
month 1-12	0.021	1.29	0.1	0.029	2.17	0.2

Panel B: OLS regression

Periods	1990-2007			1990-2010		
	$\hat{\beta}_{data}$	$\hat{t}_{data}$	$\hat{R}^2_{data}$(%)	$\hat{\beta}_{data}$	$\hat{t}_{data}$	$\hat{R}^2_{data}$(%)
VRP measure based on HAR-RV (OLS)						
month 1	0.037	0.13	2.3	-0.001	-0.003	0.0
month 1-3	0.107	2.24	6.9	0.021	0.61	0.4
month 1-6	0.140	2.14	6.1	0.047	0.83	0.9
month 1-9	0.136	1.76	3.7	0.068	1.01	1.2
month 1-12	0.114	1.16	1.8	0.070	0.93	0.9
VRP measure based on lagged RV (OLS)						
month 1	0.022	0.11	0.8	0.023	0.16	2.3
month 1-3	0.069	1.88	2.9	0.071	2.64	6.3
month 1-6	0.079	1.53	2.0	0.063	1.61	2.2
month 1-9	0.079	1.23	1.3	0.042	1.01	0.6
month 1-12	0.080	1.07	0.9	0.039	0.87	0.4

Table 11: continued

Panel C: Regression on VIX^2

Periods	1990-2007			1990-2010		
	$\hat{\beta}_{data}$	$\hat{t}_{data}$	$\hat{R}^2_{data}(\%)$	$\hat{\beta}_{data}$	$\hat{t}_{data}$	$\hat{R}^2_{data}(\%)$
month 1	0.08	1.46	1.2	-0.001	-0.13	0.0
month 1-3	0.044	1.53	2.5	0.003	0.13	0.0
month 1-6	0.066	1.52	3.0	0.007	0.20	0.0
month 1-9	0.076	1.50	2.8	0.009	0.22	0.1
month 1-12	0.081	1.49	2.5	0.010	0.23	0.1

Table 12: Model Implied Predictability of Stock Returns by Variance Risk Premium

The coefficients, t-statistics, and R^2s are imputed from the monthly VAR regression based on simulated data by LRR models. The returns are defined as the aggregate return with horizon of 1 month, 3 months, 6 months, 9 months and 12 months. The regression result is based on a 10,000-month simulation.

Periods	SVJ1F_X			SVJ1F_V			SVJ1F_X_SM			SVJ1F_V_SM			SVJ2F		
	$\hat{\beta}$	$\hat{t}$	$\hat{R}^2(\%)$	$\hat{\beta}$	$\hat{t}$	$\hat{R}^2(\%)$	$\hat{\beta}$	$\hat{t}$	$\hat{R}^2(\%)$	$\hat{\beta}$	$\hat{t}$	$\hat{R}^2(\%)$	$\hat{\beta}$	$\hat{t}$	$\hat{R}^2(\%)$
VRP measure based on expected variance															
1 month	0.081	7.36	0.6	0.060	12.07	1.5	0.062	10.81	1.3	0.044	13.94	2.0	0.042	8.42	0.8
3 month	0.191	7.88	1.2	0.133	11.84	2.5	0.156	11.34	2.9	0.105	13.92	3.9	0.097	8.92	1.5
6 month	0.268	7.92	1.3	0.186	11.57	2.5	0.240	11.27	3.6	0.155	13.59	4.3	0.136	8.90	1.5
9 month	0.297	7.89	1.1	0.208	11.39	2.1	0.283	11.14	3.5	0.178	13.29	3.9	0.150	8.84	1.3
12 month	0.309	7.87	0.9	0.217	11.27	1.8	0.306	11.02	3.2	0.189	13.08	3.4	0.156	8.80	1.1
VRP measure based on HAR-RV															
1 month	0.017	5.31	0.2	0.025	11.54	1.4	0.015	6.17	0.4	0.039	13.58	2.0	0.013	6.32	0.4
3 month	0.023	5.37	0.2	0.048	11.49	1.8	0.020	6.23	0.3	0.074	13.86	2.6	0.022	6.42	0.4
6 month	0.024	5.37	0.1	0.058	11.27	1.4	0.021	6.24	0.1	0.087	13.62	1.9	0.024	6.43	0.2
9 month	0.024	5.38	0.07	0.060	11.18	1.0	0.021	6.24	0.09	0.090	13.52	1.3	0.024	6.43	0.2
12 month	0.024	5.38	0.05	0.061	11.14	0.8	0.021	6.24	0.07	0.090	13.49	1.0	0.024	6.43	0.1
VRP measure based on lagged RV															
1 month	0.034	41.06	14.4	0.025	28.99	7.8	0.023	31.08	8.8	0.020	36.48	11.8	0.025	35.83	11.4
3 month	0.033	35.78	4.7	0.022	27.58	2.2	0.023	28.90	3.0	0.018	33.70	3.2	0.025	32.40	3.8
6 month	0.033	35.96	2.4	0.022	27.54	1.1	0.023	29.00	1.5	0.018	33.62	1.6	0.025	32.53	1.9
9 month	0.033	35.96	1.6	0.022	27.54	0.7	0.023	29.00	1.0	0.018	33.62	1.1	0.025	32.53	1.3
12 month	0.033	35.96	1.2	0.022	27.54	0.6	0.023	29.00	0.8	0.018	33.62	0.8	0.025	32.53	1.0
VIX^2															
1 month	0.019	7.36	0.6	0.016	12.07	1.5	0.020	10.82	1.3	0.016	14.00	2.0	0.015	8.60	0.8
3 month	0.045	7.88	1.2	0.036	11.84	2.5	0.051	11.34	2.9	0.039	13.96	3.9	0.036	9.03	1.7
6 month	0.064	7.92	1.3	0.051	11.57	2.5	0.079	11.27	3.7	0.058	13.63	4.5	0.053	8.99	1.9
9 month	0.071	7.89	1.1	0.057	11.39	2.1	0.093	11.14	3.6	0.067	13.33	4.1	0.060	8.91	1.7
12 month	-.074	7.87	0.9	0.059	11.27	1.8	0.100	11.02	3.3	0.072	13.10	3.6	0.063	8.85	1.4

Figure 1: log P/D ratio with and without Stock Repurchase Adjustment

This figure illustrates the log P/D ratio based on cash dividend and adjusted with repurchase. Since the repurchase data is only available since 1971, the two series are the same before 1971. The log P/D ratio denotes the log of the ratio between S&P 500 index and the total dividend (including repurchase) paid over the last 12 months.

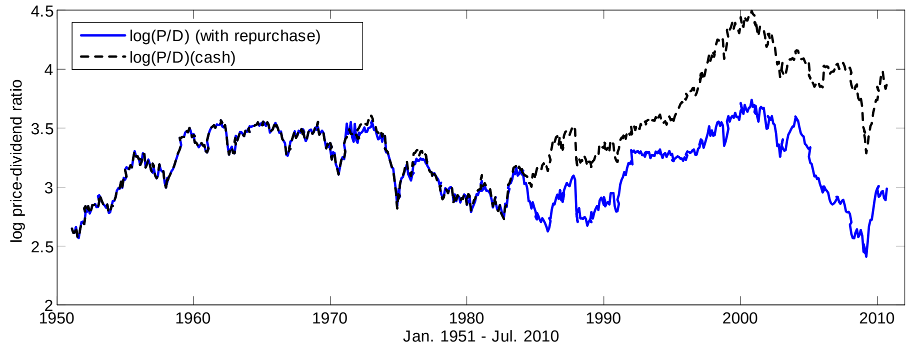

Figure 2: GDP Deflator and Consumption Price Index

This figure illustrates the quarterly GDP deflator and the consumer price index (CPI) from Q1:1951 to Q2:2010. The data is collected from the BEA